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Research Paper

FIXED POINT RESULTS IN SELF MAPPING FUNCTIONS

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The aim of this paper is to obtain some fixed point results involving occasionally weakly compatible maps in the setting of symmetric space satisfying a rational contractive condition. Our results complement, extend and unify several well known comparable results.

Keywords: Fixed point, Coincidence points, Weakly compatible

PRILIMINARIES

Definition 2.1.1: Let S and T are self maps of a metric space X. If $w = Sx = Tx$ for some $x \in X$, then x is called a coincidence point of S and T, and w is called a point of coincidence of S and T.

Definition 2.1.2: Let S and T are self maps of a metric space X, then S and T are said to be weakly compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$$

for some $x \in X$.

Definition 2.1.3: Let S and T are self maps of a metric space X, then S and T are said to be weakly compatible if they commute at their coincidence points; i.e., if $Tx = Sx$ for some $x \in X$ then $TSx = STx$.

Definition 2.1.4: Let Φ be the set of real functions

$$\phi(t_1, t_2, t_3, t_4, t_5): [0, \infty)^5 \to [0, \infty)$$

satisfying the following conditions:

- ϕ is non increasing in variables t_4 and t_5.
- There is an $h_1 > 0$ and $h_2 > 0$ such that $h = h_1 h_2 < 1$ and if $u \geq 0$ and $v \geq 0$ satisfying

a. $u \leq \phi(v, v, u, u + v, 0)$ or $u \leq \phi(v, u, v, u + v, 0)$

Then we have $u \leq h_1 v$.

And if $u \geq 0, v \geq 0$ satisfy

b. $u \leq \phi(v, v, u, 0, u + v)$ or $u \leq \phi(v, u, v, 0, u + v)$

Then we have $u \leq h_2 v$.

If $u \geq 0$ is such that

$u \leq \phi(u, 0, 0, u, u)$ or $u \leq \phi(0, u, 0, 0, u)$ or $u \leq \phi(0, 0, u, u, 0)$

Then $u = 0$.

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MAIN RESULT

Let A, B, S, T be continuous self mappings defined on the complete metric space X into itself satisfies the following conditions:

1. $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$
2. if one of $A(X)$, $B(X)$, $S(X)$, $T(X)$ is complete subspace of X.
3. The pair $\{A, S\}$ and $\{B, T\}$ are weakly compatible.
4. $d(Ax, By) \leq$

$$\alpha \frac{(d(Ax,Sx))^2 + (d(By,Ty))^2}{d(Ax,Sx)+d(By,Ty)} + \beta \frac{(d(Ax,Ty))^2 + (d(By,Sx))^2}{d(Ax,Ty)+d(By,Sx)}$$

$$+ \gamma \frac{(d(Ax,Sx))^2 + (d(Ax,Ty))^2}{d(Ax,Sx)+d(Ax,Ty)} + \delta \frac{(d(By,Sx))^2 + (d(By,Ty))^2}{d(By,Sx)+d(By,Ty)}$$

$$+ \eta \frac{d(Ax,Sx)d(By,Ty)+d(Ax,Ty)d(By,Sx)}{d(Sx,Ty)}$$

For all $x, y \in X$, $(x \neq y)$ and for non negative $\alpha, \beta, \gamma, \delta, \eta \in [0,1)$ such that $0 < 2\alpha + 2\beta + \gamma + 4\delta + \eta < 1$. then A, B, S, T have unique common fixed point in X.

Proof: For any arbitrary x_0 in X we define the sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$Ax_{2n} = Tx_{2n+1} = y_{2n}$ and $Bx_{2n+1} = Sx_{2n+2} = y_{2n+1}$

for all n = 0, 1, 2, …

On taking $y_{2n} \neq y_{2n+1}$

$$d(y_{2n}, y_{2n+1}) = d(Ax_{2n}, Bx_{2n+1})$$

From (iv) we have

$$d(Ax_{2n}, Bx_{2n+1}) \leq \alpha \frac{(d(Ax_{2n},Sx_{2n}))^2 + (d(Bx_{2n+1},Tx_{2n+1}))^2}{d(Ax_{2n},Sx_{2n})+d(Bx_{2n+1},Tx_{2n+1})}$$

$$+ \beta \frac{(d(Ax_{2n},Tx_{2n+1}))^2 + (d(Bx_{2n+1},Sx_{2n}))^2}{d(Ax_{2n},Tx_{2n+1})+d(Bx_{2n+1},Sx_{2n})}$$

$$+ \gamma \frac{(d(Ax_{2n},Sx_{2n}))^2 + (d(Ax_{2n},Tx_{2n+1}))^2}{d(Ax_{2n},Sx_{2n})+d(Ax_{2n},Tx_{2n+1})}$$

$$+ \delta \frac{(d(Bx_{2n+1},Sx_{2n}))^2 + (d(Bx_{2n+1},Tx_{2n+1}))^2}{d(Bx_{2n+1},Sx_{2n})+d(Bx_{2n+1},Tx_{2n+1})}$$

$$+ \eta \frac{d(Ax_{2n},Sx_{2n})d(Bx_{2n+1},Tx_{2n+1})+d(Ax_{2n},Tx_{2n+1})d(Bx_{2n+1},Sx_{2n})}{d(Sx_{2n},Tx_{2n+1})}$$

$$+ \beta \frac{(d(y_{2n},y_{2n}))^2 + (d(y_{2n+1},y_{2n-1}))^2}{d(y_{2n},y_{2n})+d(y_{2n+1},y_{2n-1})}$$

$$+ \gamma \frac{(d(y_{2n},y_{2n-1}))^2 + (d(y_{2n},y_{2n}))^2}{d(y_{2n},y_{2n-1})+d(y_{2n},y_{2n})}$$

$$+ \delta \frac{(d(y_{2n+1},y_{2n-1}))^2 + (d(y_{2n+1},y_{2n}))^2}{d(y_{2n+1},y_{2n-1})+d(y_{2n+1},y_{2n})}$$

$$+ \eta \frac{d(y_{2n},y_{2n-1})d(y_{2n+1},y_{2n})+d(y_{2n},y_{2n})d(y_{2n+1},y_{2n-1})}{d(y_{2n-1},y_{2n})}$$

$$d(y_{2n}, y_{2n+1}) \leq \alpha\left(d(y_{2n}, y_{2n-1}) + d(y_{2n+1}, y_{2n})\right)$$

$$+ \beta\left(d(y_{2n}, y_{2n-1}) + d(y_{2n+1}, y_{2n})\right)$$

$$+ \gamma\, d(y_{2n}, y_{2n-1})$$

$$+ \delta\left(d(y_{2n}, y_{2n-1}) + 2d(y_{2n+1}, y_{2n})\right)$$

$$+ \eta\, d(y_{2n+1}, y_{2n})$$

$$(1 - \alpha - \beta - 2\delta + \eta)d(y_{2n}, y_{2n+1}) \leq (\alpha + \beta + 2\delta + \gamma)\, d(y_{2n}, y_{2n-1})$$

$$d(y_{2n}, y_{2n+1}) \leq \frac{(\alpha + \beta + 2\delta + \gamma)}{(1 - \alpha - \beta - 2\delta + \eta)} d(y_{2n}, y_{2n-1})$$

Let us denote $\frac{(\alpha+\beta+2\delta+\gamma)}{(1-\alpha-\beta-2\delta+\eta)} = k$,

since $0 < 2\alpha + 2\beta + \gamma + 4\delta + \eta < 1$ which gives

$0 < \frac{(\alpha+\beta+2\delta+\gamma)}{(1-\alpha-\beta-2\delta+\eta)} = k < 1$ and that

$$d(y_{2n}, y_{2n+1}) \leq k\, d(y_{2n}, y_{2n-1})$$

Similarly we can show that

$$d(y_{2n}, y_{2n-1}) \leq k^2\, d(y_{2n-2}, y_{2n-1})$$

Processing the same way we can write,

$$d(y_{2n}, y_{2n-1}) \leq k^n\, d(y_0, y_1)$$

for any integer m we have

$$d(y_{2n}, y_{2n+m}) \leq d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, y_{2n+2}) + \dots\dots + d(y_{2n+m-1}, y_{2n+m})$$

$$d(y_{2n}, y_{2n+m}) \leq k^n.d(y_0, y_1) + k^{n+1}.d(y_0, y_1) + \dots\dots + k^{n+m}.d(y_0, y_1)$$

$$d(y_{2n}, y_{2n+m}) \leq k^n[1 + k + k^2 + \cdots \ldots\ldots\ldots + k^m].\, d(y_0, y_1)$$

$$d(y_{2n}, y_{2n+m}) \leq \frac{k^n}{1-k}\, .\, d(y_0, y_1)$$

as $n \to \infty$ gives that

$$d(y_{2n}, y_{2n+m}) \to 0$$

Thus $\{y_{2n}\}$ is a Cauchy sequence in X. Since $T(X)$ is complete subspace of X then the subsequence $y_{2n} = Tx_{2n+1}$ is Cauchy sequence in T(x) which converges to the some point say u in X. Let $v \in T^{-1}u$ then $Tv = u$. Since $\{y_{2n}\}$ is converges to u and hence $\{y_{2n+1}\}$ also converges to same point u.

we set $x = x_{2n}$ and $y = v$ in (iv)

$$d(Ax_{2n}, Bv) \leq \alpha \frac{(d(Ax_{2n}, Sx_{2n}))^2 + (d(Bv, Tv))^2}{d(Ax_{2n}, Sx_{2n}) + d(Bv, Tv)}$$

$$+ \beta \frac{(d(Ax_{2n}, Tv))^2 + (d(Bv, Sx_{2n}))^2}{d(Ax_{2n}, Tv) + d(Bv, Sx_{2n})}$$

$$+ \gamma \frac{(d(Ax_{2n}, Sx_{2n}))^2 + (d(Ax_{2n}, Tv))^2}{d(Ax_{2n}, Sx_{2n}) + d(Ax_{2n}, Tv)}$$

$$+ \delta \frac{(d(Bv, Sx_{2n}))^2 + (d(Bx_{2n+1}, Tv))^2}{d(Bv, Sx_{2n}) + d(Bx_{2n+1}, Tv)}$$

$$+ \eta \frac{d(Ax_{2n}, Sx_{2n})d(Bv, Tv) + d(Ax_{2n}, Tv)d(Bv, Sx_{2n})}{d(Sx_{2n}, Tv)}$$

as $n \to \infty$

$$d(u, Bv) \leq (\alpha + \beta + \gamma + \delta + \eta)\; d(u, Bv)$$

which contradiction

implies that $Bv = u$ also $B(X) \subset S(X)$ so $Bv = u$ implies that $u \in S(X)$.

Let $w \in S^{-1}(X)$ then $w = u$ setting $x = w$ and $y = x_{2n+1}$ in (iv) we get

$$d(Aw, Bx_{2n+1}) \leq \alpha \frac{(d(Aw, Sw))^2 + (d(Bx_{2n+1}, Tx_{2n+1}))^2}{d(Aw, Sw) + d(Bx_{2n+1}, Tx_{2n+1})}$$

$$+ \beta \frac{(d(Aw, Tx_{2n+1}))^2 + (d(Bx_{2n+1}, Sw))^2}{d(Aw, Tx_{2n+1}) + d(Bx_{2n+1}, Sw)}$$

$$+ \gamma \frac{(d(Aw, Sw))^2 + (d(Aw, Tx_{2n+1}))^2}{d(Aw, Sw) + d(Aw, Tx_{2n+1})}$$

$$+ \delta \frac{(d(Bx_{2n+1}, Sw))^2 + (d(Bx_{2n+1}, Tx_{2n+1}))^2}{d(Bx_{2n+1}, Sw) + d(Bx_{2n+1}, Tx_{2n+1})}$$

$$+ \eta \frac{d(Aw, Sw)d(Bx_{2n+1}, Tx_{2n+1}) + d(Aw, Tx_{2n+1})d(Bx_{2n+1}, Sw)}{d(Sw, Tx_{2n+1})}$$

as $n \to \infty$

$$d(Aw, u) \leq (\beta + \gamma + \delta + \eta)\; d(Aw, u)$$

which contradiction

implies that, $Aw = u$ this means $Aw = Sw = Bv = Tv = u$.

since $Bv = Tv = u$ so by weak compatibility of (B, T) it follows that, $BTv = TBv$ and so we get

$$Bu = BTv = TBv = Tu.$$

Since $Aw = Sw = u$ so by weak compatibility of (A, S) it follows that $SAw = ASw$ and so we get

$$Au = ASw = SAw = Su.$$

Thus from (iv) we have

$$d(Aw, Bu) \leq \alpha \frac{(d(Aw, Sw))^2 + (d(Bu, Tu))^2}{d(Aw, Sw) + d(Bu, Tu)}$$

$$+ \beta \frac{(d(Aw, Tu))^2 + (d(Bu, Sw))^2}{d(Aw, Tu) + d(Bu, Sw)}$$

$$+ \gamma \frac{(d(Aw, Sw))^2 + (d(Aw, Tu))^2}{d(Aw, Sw) + d(Aw, Tu)}$$

$$+ \delta \frac{(d(Bu, Sw))^2 + (d(Bu, Tu))^2}{d(Bu, Sw) + d(Bu, Tu)}$$

$$+ \eta \frac{d(Aw, Sw)d(Bu, Tu) + d(Aw, Tu)d(Bu, Sw)}{d(Sw, Tu)}$$

$$d(u, Bu) \leq (\alpha + \beta + \delta)d(u, Bu)$$

which contradiction

implies that $Bu = u$.

Similarly we can show $Au = u$ by using (iv). Therefore

$$u = Au = Bu = Su = Tu.$$

Hence the point u is common fixed point of A, B, S, T.

If we assume that $S(X)$ is complete then the argument analogue to the previous completeness argument proves the theorem. If $A(X)$ is complete then $u \in A(X) \subset T(X)$. similarly if $B(X)$ is complete then $u \in B(X) \subset S(X)$ This complete prove of the theorem.

Uniqueness: Let us assume that z is another fixed point of A, B, S, T in X different from u. $i.e.\ u \neq z$ then

$d(u, z) = d(Au, Bz)$

from (iv) we get

$d(u, z) \leq (\beta + \gamma + \delta + \eta) d(u, z)$

which contradiction the hypothesis .

Hence u is unique common fixed point of A, B, S, T in X.

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