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Research Paper

ANALYSIS OF NON STATIONARY SIGNALS
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This paper presents the analysis of Cohen's class quadratic non stationary signals by Stockwell transform. Stockwell transform (S-Transform) is an improved version for time frequency representation (TFR'S) of signals when compared with Short Time Fourier Transform (STFT), Continues Wavelet Transform (CWT), and Wigner Ville Distribution (WVD), etc. Hence S-Transform gains more popularity in now a days. In this paper a set of classical Cohen's type quadratic signals are considered and performed TFR'S with respect to S-Transform. Further these signals are tested under white gaussian noise environment and evaluated the mean square error. The results show that S-Transform gives better time frequency resolution at multiple frequency components level when compared with other TFR'S.

Keywords: Stockwell transform, Quadratic signals, Wavelet, Wigner ville

INTRODUCTION

Fourier Transform (FT) is applicable only for periodic signals and it cannot give information simultaneously about spectral density components in terms of time-frequency plane (Bracewell, 1965). Also Fourier transform provides information only in frequency domain but not in time domain. To overcome these disadvantages Time Frequency Representations (TFR'S) have gained more importance in analysis of non stationary signals (Boashash, 2003). Many TFR'S have been introduced earlier and are classified into two types i.e., Linear and Quadratic TFR'S. Short Time Fourier Transform (STFT) and

Continues Wavelet Transform (CWT), Stockwell Transform (S-Transform) are most commonly used linear TFR'S while Wigner Ville Distribution (WVD), Ambiguity Function, Gabor Function, Cohen's class etc are quadratic types (Cohen Leon, 1989; and Torres-Cisneros *et al.*, 2006).

STFT is a simple Windowed Fourier Transform (WFT) defined for its window limits. STFT is computed by taking inner product of the signal with a time frequency localized window function. The fixed window width function utilized in STFT results in poor time-frequency resolution (Djurovic Igor *et al.*, 2008). Similarly for CWT although window is variable scaling function, at

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larger window scaling ratios results in poor resolution at lower frequency contents and lower window scaling ratios results in poor resolution at higher frequency contents (Assous Said and Boualem Boashash, 2012). To overcome these disadvantages lead to the development of a new method for non stationary signal analysis called Stockwell Transform (Stockwell and Robert Glenn, 2007). S-Transform defined by Stockwell is a hybrid version of STFT and CWT. The major advantage of S-Transform is it preserves phase information which was faded in CWT and STFT. S-Transform uses gaussian window for its time frequency computations (Ventosa Sergi *et al.*, 2008). The advantage of gaussian window is its shape doesn't change during transform which can still preserves information about centre frequency content and phase information (Stockwell Robert Glenn *et al.*, 1996). Many Quadratic Time Frequency Representations (QTFR'S) like WVD, Gabor, and Spectrogram etc are also uses mother wavelets of gaussian function as a window for its TFR'S. But the QTFR'S suffers from cross terms existences which results in poor time-frequency resolution (Hlawatsch and Boudreaux-Bartels, 1992).

ANALYTICAL THEORY

Short Time Fourier Transform

STFT of a signal $f(t)$ for the window function $\phi(t)$ at the point (b, ξ) is defined as

$$STFT\{f(t)\} = F_{\phi} f(b, \xi) = \int_{-\infty}^{\infty} f(t) \overline{\phi_{b, \xi}(t)} dt; \dots(1)$$

where

$$\phi_{b, \xi}(t) = \phi(t - b) e^{j \xi t} \dots(2)$$

For inverse STFT has to satisfy the following condition, i.e.,

$$\phi(0) = \int_{-\infty}^{\infty} \phi(t) dt \neq 0; \dots(3)$$

The function $\phi_{b, \xi}(t)$ acts as window interval and can be considered as kernel function. The signal is split into different parts of equal length and each part is projected with the smooth window function which nearly vanished at the edges. If the window function is sufficiently narrow with respect to extracted input signal then its corresponding Fourier transform can be obtained easily. But it cannot be happened more often since input signal is non stationary. So STFT may be interpreted as the components of the function $f(t)$ with respect to the window plane. To resolve the components in the frequency axis and in time axis, it is necessary to compute STFT every time due to fixed window length. So computation complexity exists in STFT (Goswami Jaideva and Andrew Chan, 2011) (Chapter 4), (Portnoff Michael, 1980).

Continues Wavelet Transform

It is also called as Integral Wavelet Transform (IWT) and is defined as:

$$CWT\{f(t)\} = F_{\phi} f(b, a) = \int_{-\infty}^{\infty} f(t) \overline{\phi_{b, a}(t)} dt; \dots(4)$$

where

$$\phi_{b, a}(t) = a^{-1/2} \phi \left[\frac{t-b}{a} \right]; a > 0; \dots(5)$$

b = Translation parameter

a = Dilation parameter

$a^{-1/2}$ = Normalization factor

' $f(t)$ ' can be recovered from CWT if ' $\phi(t)$ ' satisfied by following condition

$$\overline{\phi(0)} = \int_{-\infty}^{\infty} \phi(t) dt = 0; \dots(6)$$

CWT is a complex valued function in terms of both position and scale of a window function. In this transform, matching of window with input signal is incorporated. When the window function matches with shape of input signal for a specific

scale and location then a large transform coefficient value is obtained. However if window doesn't matches with shape of input signal then a low value of transform coefficient is obtained. In this way obtained all coefficients are plotted in the two dimensional time frequency plane. Thus filling the entire time frequency plane in a smooth and continuous manner can be called as Continues Wavelet Transform (CWT). Scaling technique is used to complete CWT. Dilation parameter 'a' determines the scaling level, for 'a>1' window function will become wider along time axis. Similarly for '0<a<1' window function become smaller along time axis. But the major problem associated with CWT is when dilation parameter is very high then ratio of (t/a) is low, which falls under narrow frequency axis, results in poor time-frequency resolution. In addition to this existed unsymmetrical nature of many wavelets which are acts as window function, CWT lacks to preserve phase information which is a major drawback in CWT (Goswami Jaideva and Andrew Chan, 2011) (Chapter 4), (Rioul Olivier and Martin Vetterli, 1991).

Stockwell Transform

Stockwell transform (Stockwell *et al.*, 1996) for $x(t)$ is defined as time frequency transform of product $x(t)$ and gaussian window function.

$$S(t, f) = \int_{-\infty}^{\infty} x(\tau) w(t - \tau, f) e^{-j2\pi f\tau} d\tau; \quad \dots(7)$$

where

$$w(t - \tau, f) = \frac{1}{\sigma(f)\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2\sigma(f)^2}} \quad \dots(8)$$

Standard deviation

$$\sigma(f) = \frac{1}{|f|} \quad \dots(9)$$

Stockwell transform uses gaussian function as window to obtain time interval selection on $f(t)$.

The window function is time localized with a shift of 'τ' with respect to input signal. Similarly kernel $e^{-j2\pi f\tau}$ is to select which frequency has to be localized within the input signal. Due to stationary kernel it retrieves the phase information and can be called as absolute referenced phase information. The extraction of globally referenced phase feature is easier due to no cross term existences. Stockwell Transform produces better low frequency domain analysis because its window is directly proportional to input frequency (8). Similarly it produces better time domain analysis at high frequency contents. Moreover computation complexity of S-Transform is less when compared with other linear time-frequency transform techniques because its coefficients are conjugate symmetric for the analysis of non stationary input signal.

The major advantage of S-Transform is it uses gaussian integral functionality property which produces frequency invariant amplitude response and can preserve phase information as well as information about centre frequency content by considering Gaussian as unit area localizing function. Multi resolution phenomenon can be obtained in S-Transform by varying frequency with respect to window width. Hence S-Transform provides better time frequency resolution when compared with other linear time-frequency transform techniques like STFT, CWT, etc. (Mansinha *et al.*, 1997; and Adams Michael *et al.*, 2002).

Wigner Ville and Ambiguity Function

Both Wigner Ville distribution and Ambiguity Function are QTFR'S used majorly when time frequency energy distribution is needed. These functions distribute the instantaneous signal power over entire time frequency plane.

Wigner Distribution (WD) is defined as follows:

$$W_x(t, f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau; \quad \dots(10)$$

$x^*(t)$ Complex conjugative of $x(t)$

Wigner Distribution is performed on the basis of autocorrelation of with specified equal amount of lead and lag phase shift $\frac{\tau}{2}$. WD computation requires a lead phase shift of input signal $x(t)$ which is non casual operation. Due to this non casual operation it is not possible to perform Wigner distribution until $x(t)$ is known for all instantaneous time values. As there is no separate window function associated with WD unlike other wavelet transforms, kernel $e^{-j2\pi f\tau}$ is used to localize the frequency of instantaneous auto correlation product $x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right)$. When the kernel $e^{-j2\pi f\tau}$ is projected on instantaneous autocorrelation product it produces both positive and negative coefficients. The negative coefficients represent negative energy density which has no sense physically. This is the major drawback in WVD. But Wigner distribution produces joint time-frequency moments which are helpful in analysis of non stationary signal interms of instantaneous power, instantaneous frequency and group delay (Boudreaux-Bartels *et al.*, 1986; and Boashash Boualem, 1988).

Also Ambiguity Function (AF) given as below

$$\chi(\tau, f) = \int_{-\infty}^{\infty} x(t)x^*(t - \tau) e^{-j2\pi f\tau} dt; \quad \dots(11)$$

$x^*(t)$ Complex conjugative of $x(t)$

Ambiguity function is defined as two variable representations of autocorrelation functions. Ambiguity function can be performed as Fourier transform for inner product of input signal and shifted version of complex conjugate input signal it's self. The shifted time quantity can be called

as Doppler Shift (τ). This Doppler shift plays vital role in determining the ambiguity nature of ' $\chi(\tau, f)$ ' in analysis of different QTFR'S. In general ambiguity function is divided into two categories, i.e., Narrow band Ambiguity function and Wide band Ambiguity function. When the Doppler shift of a reflected echo non stationary signal is in comparable range to that of light velocity then ambiguity function falls under the category of Narrow band ambiguity otherwise it is a wide band ambiguity. All the QTFR'S can be estimated as product of ambiguity function and ambiguity window kernel (Stein Seymour, 1981; and Costas John, 1984).

Even though QTFR'S are better in analysis of non stationary signal, every Quadratic time frequency transform has its limitations. For example Wigner Ville distribution provides better quadratic time frequency distribution but it suffers from effect of cross terms existences. Similarly Ambiguity function is also suffers in prediction of Doppler shift which results in high probability error during non stationary signal time-frequency analysis. Hence in many cases Stockwell Transform is most suitable window transform technique in analysis of non stationary signals (Stockwell, 2007).

RELATED WORK

The fundamental challenge of different TFR'S is adoption of suitable window to achieve good tradeoff between time and frequency analysis. Various time-frequency transforms are introduced so far in which basic linear transforms are STFT and CWT. Short Time Fourier Transform was introduced by Gabor in 1946 to measure the frequency variation of speech signal with respect to time. Basic idea of STFT is dividing signal into small components

and applying Fourier transform to individual components to obtain frequency contents with respect to time. Similarly Continuous Wavelet Transform was introduced by Morlet in 1984 which eliminates some disadvantages of STFT by using scaling window. During 1986 Stephane Mallat and Yves Meyer developed the multi resolution technology by introducing Discrete Wavelet Transform (DWT) (Shensa Mark, 1992). Wavelet Transform has gained more popularity in analysis of non stationary signals after the introduction of different orthogonal wavelets by Ingrid Daubechies in 1987 (Daubechies Ingrid, 1990). In 1993 David Edward Newland proposed the Harmonic wavelet transform which is migrated version of STFT and CWT (Newland David, 1993; and Sridhar *et al.*, 2014).

Stockwell Transform introduced by Stockwell in 1997 has made dramatic changes in analysis of non stationary signals. Advantages of S-Transform with respect to linear and bilinear transforms are mention earlier in this paper Section II. But limitation to Stockwell transform is its redundancy nature during time-frequency analysis.

Many QTFR'S earlier works like Smoothed pseudo WD, Choi-Williams distribution, Cone-Kernel distribution and Rihaczek distributions are introduced based on Wigner Ville Distribution (Papandreou-Suppappola Antonia and Seth Suppappola, 2002). These distributions are simply modified kernel versions of Wigner Ville Distribution (WVD) introduced by Wigner in 1932 later developed by Ville in 1948. Smoothed pseudo WD (PSWVD) reduces the interference terms problem existed in WVD (Papandreou-Suppappola Antonia, 1995). PSWVD is given as below.

$$PSW_x(t, f) = \int_{-\infty}^{\infty} q(t - \tau) \int_{-\infty}^{\infty} w\left(\frac{\tau}{2}\right) w^*\left(\frac{\tau}{2}\right) x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau; \dots(12)$$

$q(t - \tau)$ = Low pass function.

The above Equation (12) represents PSWVD which provides short time WD and improves instantaneous time smooth using low pass function (Yan Yong-Sheng *et al.*, 2005). However a finite amount of reduced interference distributions can also be obtained using Choi-Williams distribution (CW). Choi-Williams distribution is given as

$$CW_x(t, f) = \iint_{-\infty}^{\infty} W_x(\eta, \tau) \phi(\eta, \tau) e^{j2\pi(\eta t - \tau f)} d\eta d\tau; \dots(13)$$

where η = Variable parameter on time axis

τ = Variable parameter on frequency axis

W_x = Wigner Distribution

$$\phi(\eta, \tau) = \exp(-\alpha[(\eta\tau)^2]) \dots(14)$$

Choi-Williams Distribution implements the exponential kernel function which reduces cross term products during time frequency analysis. Parameter ' α ' is used to vary the shape of exponential function. ' α ' is optimized in accordance with shape of non stationary signal (Papandreou-Suppappola Antonia and Faye Boudreaux-Bartels, 1993). Similarly Cone shaped distribution function is also used to reduce some extent of interference terms. The only difference between Choi-Williams and Cone shaped is kernel function. Kernel for Cone shaped distribution is given as

$$\phi(\eta, \tau) = \frac{\sin(\pi\eta\tau)}{\pi\eta\tau} \exp(-2\pi\alpha[\tau^2]) \dots(15)$$

Kernel function in (t, τ) parameters is defined as

$$\phi(t, \tau) = \begin{cases} \frac{1}{\tau} \exp(-2\pi\alpha[\tau^2]), & 2|t| \leq |\tau|; \\ 0 & \text{otherwise}; \dots(16) \end{cases}$$

The above cone shaped kernel approach (13) significantly reduces cross terms in case of speech signals which consists of different complex sinusoidal exploits and apparent stationary instantaneous frequencies (Zhao Yunxin *et al.*, 1990).

Kernel function for Rihaczek distributions is given as:

$$\phi(\eta, \tau) = \exp\left(-j2\pi\left(\frac{\eta\tau}{2}\right)\right) \dots(17)$$

Kernel used in Rihaczek distribution has greatest extent to satisfy the marginal distribution properties during time-frequency analysis. However Rihaczek distribution is also falls under limitation in estimation of instantaneous phase synchrony with respect to time- frequency analysis. Due to this limitation its time-frequency concentration is low. In this way all bilinear transforms has its advantages as well as limitations. Hence different kernels have to be chosen for different non stationary signals to get compatibility between window function and input signal. Choice of kernel distribution is varied from one signal to other signal for better analysis because nature of signal is unknown (Jeong Jechang and William Williams, 1992; Chen Victor and Hao Ling, 2001; and Scharf Louis *et al.*, 2005).

RESULTS AND DISCUSSION

In this section three different quadratic Cohen's class equations are considered and performed TFR'S (Flandrin Patrick, 1984).

Case 1: Consider following signal as given below

$$x(t) = \cos(2\pi \sin(5\pi t) + 120\pi t) + \cos(168\pi t + 28\pi t^2);$$

$$0 \leq t < 1; \dots(18)$$

Equation (18) consists of two components in which one component is phase modulated signal and other component constitutes to chirp signal.

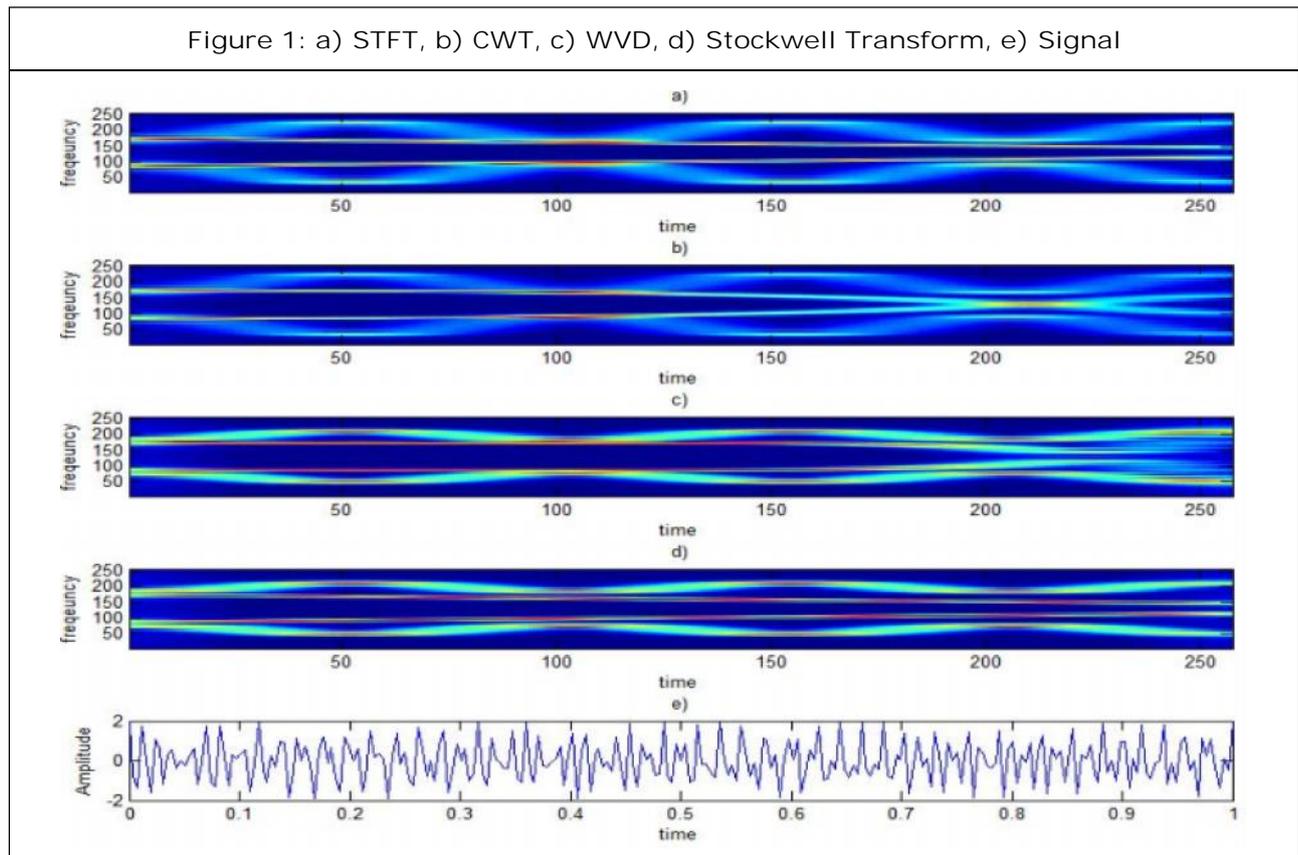
The sampling period is taken as $T_s = \frac{1}{512} \text{ sec};$

Standard deviation $\sigma = \left(\frac{n}{256}, 1 \leq n \leq 256\right);$

where $t = n \Delta t; n = 0,1,2, \dots, 512.$

Figure 1 shows various TFR'S comparison for which time axis is in (ms) and frequency axis is in (Hz). By observation it can be interpreted that time-frequency plane produces chirp components enclosed by modulated envelope components. For the frequency range ' $f' = 0 \text{ to } 150 \text{ Hz}$ ' time-frequency plane gives lower chirp component and ' $f' = 150 \text{ to } 250 \text{ Hz}$ ' gives upper chirp component with respect to instantaneous time duration. By fixed sampling frequency 'a) STFT' TFR produced the poor time-frequency resolution to entire duration of signal ' $0 \leq t < 1$ '. At different time instantaneous values of ' Δt ' from 0.2 ms to 0.25 ms 'b) CWT' indulges in overlap of chirp components. This is due to unable to retrieve the phase information at higher scaling ratios. Though 'c) WVD' produced the comparative time-frequency resolution with respect to 'd) Stockwell Transform', at multiple frequency component stages of a signal $x(t)$ leads to occurrence of interference terms results in poor time-frequency resolution.

Finally from Figure 1 it can be concluded that STFT unable to get phased information for modulated signal due to fixed window length proportions, results in poor time frequency resolution. Similarly CWT also suffers in poor time



frequency resolution for modulated component. Indeed, as scaling ratio increases toward time axis, CWT gets narrow time frequency resolution which is shown in Figure 1. Even though Wigner distribution is able to get resolution for both chirp and modulated signal, due to cross term existences WD gets the faded time frequency resolution at low frequency contents. When comparing S-Transform with all TFR'S, it gives better time frequency resolution since it preserves both frequency and phase information (Azam Samiul *et al.*, 2014).

Case 2: Consider following signal as given below

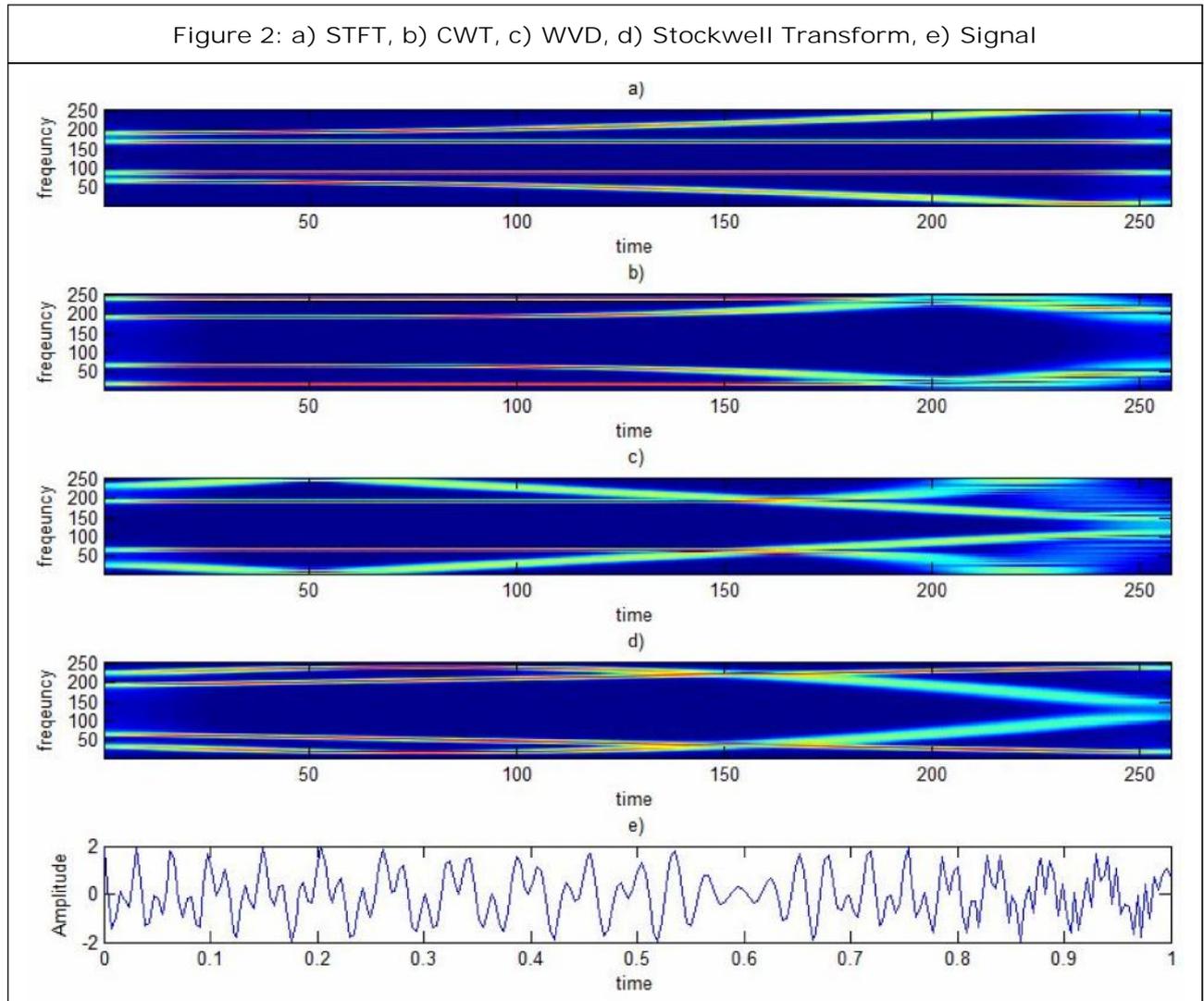
$$y(t) = \cos(144\pi(t - 0.3)^2 + 30\pi t) + \cos(128\pi t - 50\pi t^2);$$

$$0 \leq t < 1; \quad \dots(19)$$

Equation (19) constitutes to one chirp component and one hyperbolic component.

Figure 2 shows TFR'S for the signal $y(t)$ in which signal consists of rapid variation in ripple frequency contents from the time duration ' $t = 0.7$ sec. Lower components of both chirp and hyperbolic function are associated in the frequency range of ' $f = 0$ to 150 Hz. Whereas higher components are in the range of ' $f = 150$ to 250 Hz TFR of STFT completely lost the phase direction of hyperbolic component from ' $\Delta t = 0.15$ ms. This is due to fixed window length. Similarly CWT also lost its hyperbolic TFR view since it is unable to preserve progressive phase information. But in case of WVD even though better retrieval phase information is obtained, as signal enters in ripple frequency contents at ' $t = 0.7$ sec many interference terms are exited during computation results in poor time-frequency resolution.

Finally from Figure 2 it can be concluded that all the transforms produces similar time



frequency resolution at initial time duration of signal $y(t)$. But as $y(t)$ depicts in high frequency contents with respect to time axis, then STFT, CWT and WD are unable to produce better time frequency resolution when compared with standard S-Transform as shown in Figure 2.

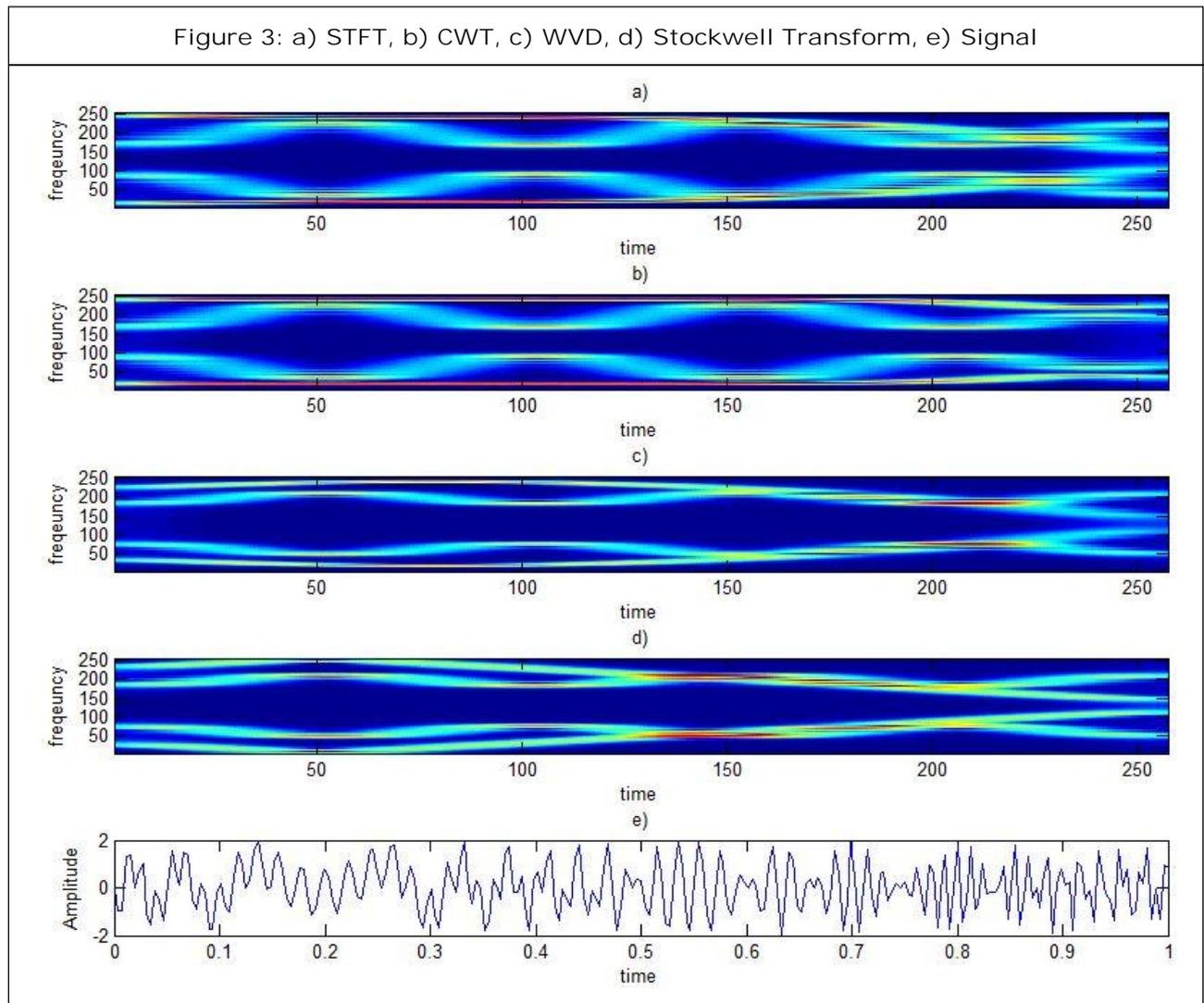
$$z(t) = \cos(2\pi \sin(5\pi t) + 120\pi t) + \cos(144\pi(t - 0.3)^3 + 30\pi t); \dots(20)$$

Case 3: Consider following signal as given below

Equation (21) consists of phase modulated component and hyperbolic component.

TFR'S view of signal $z(t)$ can be observed from the Fig.3. In similar fashion to signal $y(t)$, $z(t)$ also consists of multiple frequency component variations for the durations of ' Δt ' from '0 to 0.5 sec', ' Δt_1 ' from '0.5 to 0.8' and ' Δt_2 ' from '0.8 to 1 sec'. Envelope of a phase modulated component is enclosed by upper and lower components of hyperbolic function as shown in Figure 3.

Both 'a) STFT' and 'b) CWT' losses its phase information and 'c) WVD' suffers from cross term existence results in poor time-frequency localization. Finally from Figure 3 it can be



interpreted that out of all TFR'S Stockwell Transform produced the better enhancement in time-frequency localization due to preserve of phase and instantaneous frequency.

Finally effect of White Guassian noise on various TFR'S is evaluated by taking Linear Frequency Modulation (LFM) signal (Barbarossa Sergio, 1995; and Djuric Petar, 1996).

Consider

$$f(t) = \sin(2\pi t + 4\pi \cos(4\pi t)); \quad \dots(21)$$

Initially signal $f(t)$ is mixed with white gaussian noise and Signal to Noise Ratio (SNR) is performed. Finally Mean Square Error (MSE) is

evaluated for instantaneous frequency estimation of $f(t)$ interms of various SNR values (Ephraim Yariv and David Malah, 1984; Boashash, Boualem, 1992; and Katkovnik Vladimir and Ljubiša Stankovic, 1998).

Instantaneous frequency for $f(t)$ is given as

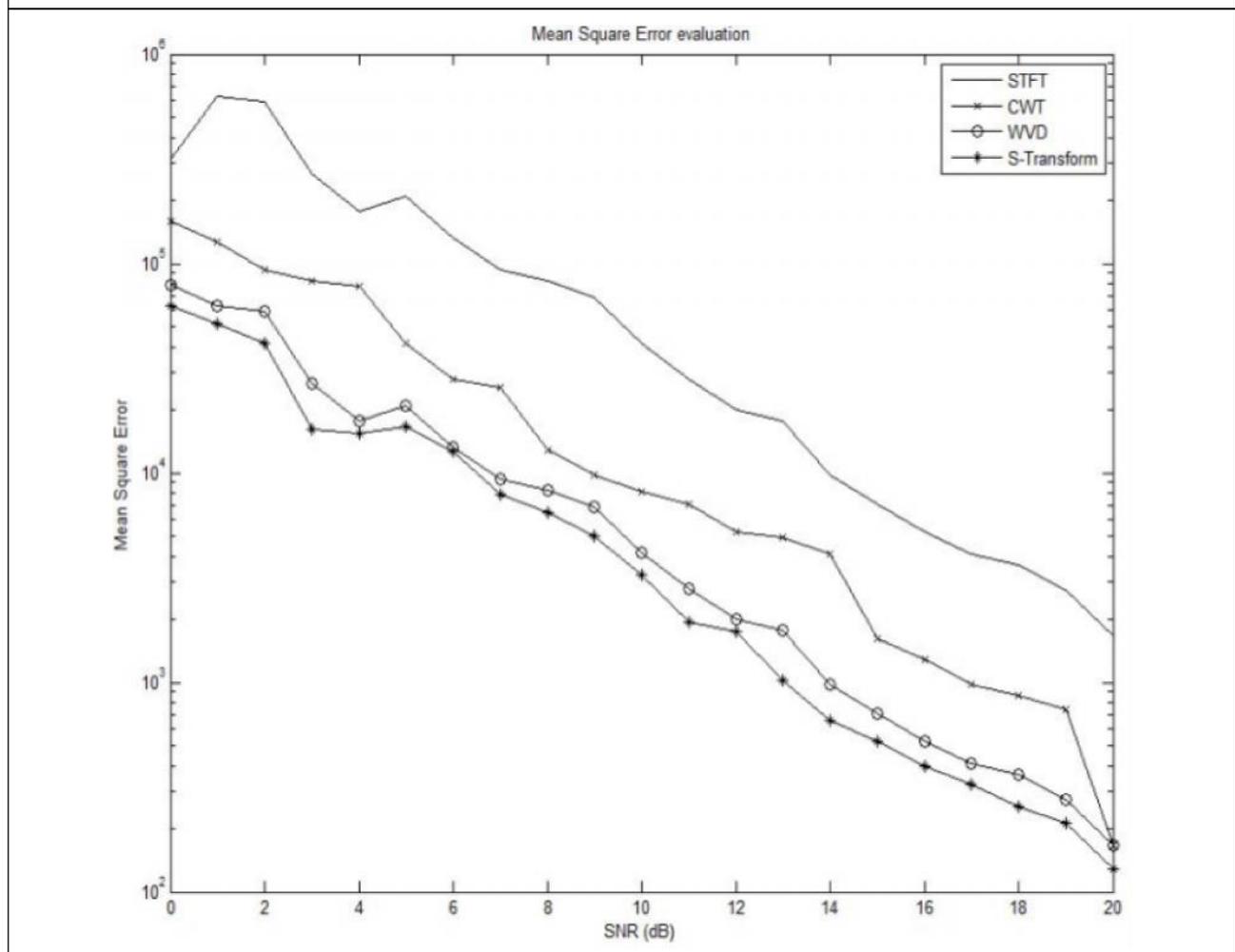
$$\Delta f_i = 8\pi \sin(4\pi t); \quad \dots(22)$$

$$SNR = 10 \log_{10} \left(\frac{1}{\sigma_n^2} \right); \quad \dots(23)$$

where σ_n^2 = Variance of noise.

From Figure 4 it can be concluded that S-Transform has least MSE when compared with other transforms.

Figure 4: Mean Square Error Comparison for Various TFR'S



CONCLUSION

In this paper Cohen’s class quadratic signals are tested on various time frequency transforms. Out of all the TFR’S, S-Transform has better time frequency resolution when compared with the other TFR’S. It can be observed that, though S-Transform comes under the category of linear transform, it has ability to analyze quadratic signals better than other QTFR’S. Moreover S-Transform produces least mean square error in comparison to other TFR’S. But in certain cases QTFR’S produces better time-frequency localization when compared with S-Transform. This is possible only when QTFR’S kernels are

well suited for unknown non stationary nature of input signal. Limitation to this analysis is redundancy behavior of S-Transform.

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