

Research Paper

# INVESTIGATION OF STRESSES IN BI-MATERIAL STRIP SUBJECTED TO CONSTANT TEMPERATURE AND TEMPERATURE GRADIENT BY FEM

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Stresses and deflections in a bi-material strip subjected to temperature gradient that varies linearly in longitudinal direction are studied using FE analysis. The comparison with constant thermal loading is done and the results are plotted and conclusions are drawn showing the effect of temperature gradient.

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Keywords: Bi-material strip, Axial temperature gradient, FEM, Thermal loading

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## INTRODUCTION

Bi-material strip consist of two metals with different coefficients of thermal expansion bonded together. As the temperature varies from the temperature at which the metals were bonded, the metals expand by different amounts and the composite experiences a shearing force.

A bi-material strip is used to convert a temperature change into mechanical displacement. The strip consists of two strips of different metals which expand at different rates as they are heated, usually steel and copper, or in some cases brass instead of copper. The strips are joined together throughout their length by riveting, brazing or welding. The different expansions force the flat strip to bend one way if heated, and in the opposite direction if cooled

below its initial temperature. The metal with the higher coefficient of thermal expansion is on the outer side of the curve when the strip is heated and on the inner side when cooled.

The sideways displacement of the strip is much larger than the small lengthways expansion in either of the two metals. This effect is used in a range of mechanical and electrical devices. In some applications the bimetal strip is used in the flat form. In others, it is wrapped into a coil for compactness. The greater length of the coiled version gives improved sensitivity..

In present work the stresses produced are investigated in bi-material strip subjected to constant temperature and temperature gradient which varies linearly in the longitudinal direction by finite element analysis using ANSYS software.

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## DESIGN PARAMETER AND MATERIAL

The Bi-material strip made of steel and brass with equal length, breadth and thickness subjected to constant temperature and axial temperature gradient is considered. The FE analysis is carried out by using following specifications.

- Constant Load Temp = 50 °C
- Axial temperature gradient = 0 to 100 °C
- Length of Bi-material strip L = 100 mm
- Width of steel plate b = 10 mm
- Thickness of steel plate t = 10 mm
- Width of brass plate b = 10 mm
- Thickness of brass plate t = 10 mm

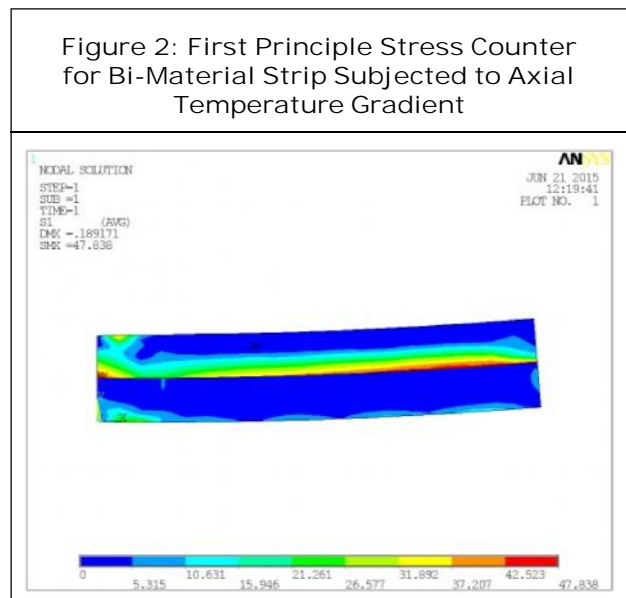
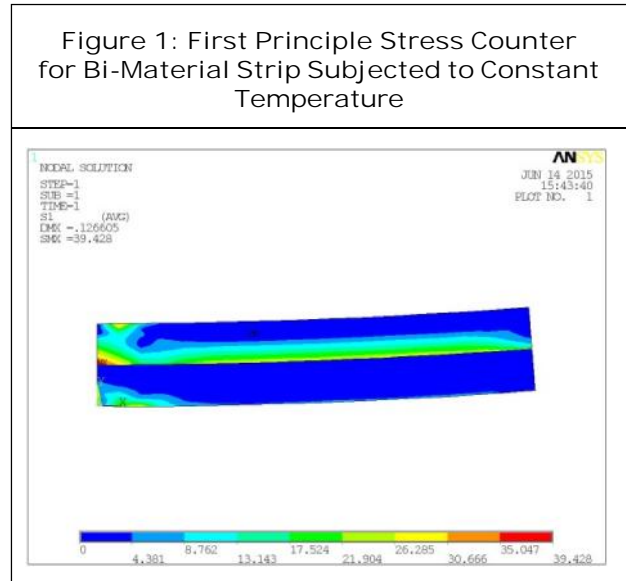
The design parameters of bi-material strip are shown in Table 1.

Table 1: Design Parameters of Bi-Material Strip Combination		
Parameter	Steel	Brass
Young's Modulus E (N/mm <sup>2</sup> )	2 x 10 <sup>5</sup>	1 x 10 <sup>5</sup>
Coefficient of Thermal Expansion (/°C)	11 x 10 <sup>-6</sup>	16.5 x 10 <sup>-6</sup>
Poisson's Ratio	0.3	0.15

## METHODOLOGIES

### Finite Element Analysis

For the analysis purpose 2-D models are prepared using ANSYS. The FE analysis is carried out with 4-node PLANE42 2D structural solid element, by applying constant temperature and axial temperature gradient. The materials considered are Steel and Brass. FE results are compared with analytical results. The properties of material are provided and mesh model is developed. After solving, the first principle stress contour at nodal region is shown in Figures 1 and 2.



### Analytical Analysis

The analytical equations and analysis are presented as follows.

$$e_b = \text{strain in brass}$$

$$e_s = \text{strain in steel}$$

$$e_b + e_s = \Delta T(\alpha_b - \alpha_s)$$

$$\sigma_b/E_b + \sigma_s/E_s = \Delta T(\alpha_b - \alpha_s)$$

$$F/(A \times E_b) + F/(A \times E_s) = \Delta T(\alpha_b - \alpha_s)$$

$$F = \Delta T(\alpha_b - \alpha_s)/((1/A \times E_b) + (1/A \times E_s))$$

Table 2: Comparison of Stresses and Deflection for the Bi-material Strip Subjected to Constant Temperature at Different Location Along Length

Distance from Support (mm)	FE Stresses (N/mm <sup>2</sup> )				Analytical Stress (N/mm <sup>2</sup> )
	S <sub>x</sub>	S1	S2	S3	
0	39.372	39.428	0	123.13	18.33
25	24.2	24.201	0.72	23.849	18.33
50	25.012	25.012	0	25.05	18.33
75	24.977	24.977	0.115	24.92	18.33
100	21.346	23.192	0	28.065	18.33

Figure 3: Variation in Stresses Along Length of Bi-Material Strip Subjected to Constant Temperature

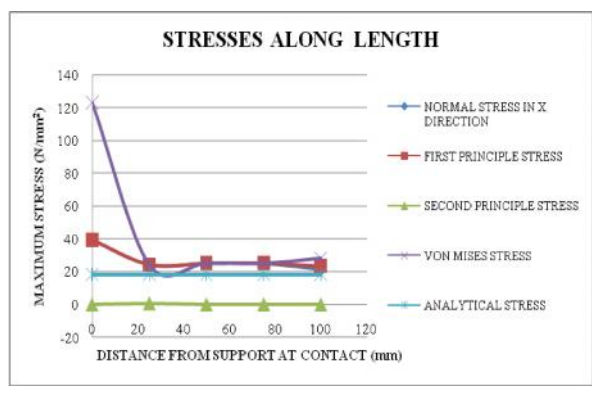


Table 3: Comparison of Stresses for the Bi-Material Strip Subjected to Axial Temperature Gradient

Distance from Support (mm)	Stresses (N/mm <sup>2</sup> )			
	S <sub>x</sub>	S1	S2	S3
0	47.77	47.84	0	148
25	33.297	33.306	0	34.1
50	40.017	40.019	0	40.030
75	45.702	45.706	2.57	44.474
100	42.406	46.136	0	55.443

$$F = 50 (16.5 - 11) \times 10^{-6} / ((1/100 \times 1 \times 10^5) + (1/100 \times 2 \times 10^5))$$

$$F = 1833.33 \text{ N}$$

$$\sigma_s = F/A$$

$$\sigma_s = 1833.33/100$$

$$\sigma_s = 18.33 \text{ N/mm}^2$$

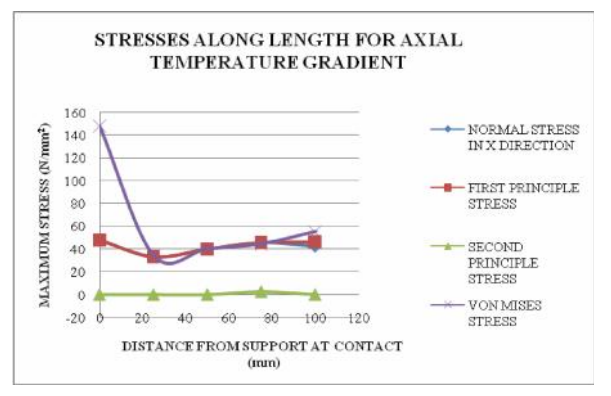
$$\delta = (e_s + \alpha_s \times \Delta T) \times L$$

$$\delta = 0.010 \text{ mm}$$

### COMPARISON OF RESULTS

To study the effect on stresses in Bi-material strip subjected to constant temperature, the FE and analytical stresses are tabulated in Table 2. and the stresses in bi-material strip subjected to axial temperature gradient are tabulated in Table 3. For the analysis purpose the FE stresses in a bi-material strip are studied at a distance of 0 mm, 25 mm, 50 mm, 75 mm and 100 mm from fixed support.

Figure 4: Variation in Stresses Along Length for Bi-Material Strip Subjected to Axial Temperature Gradient



### DISCUSSION

Though the detailed results are presented earlier here an attempt is made to compare the

stresses obtained in bi-material strip subjected to constant temperature and axial temperature gradient. For comparison of stresses first principle stress is considered. The detailed discussion is as follows.

### **Comparison of Analytically Calculated Stresses with FE Analysis**

The initial study on stresses along length carried out on the bi-material strip subjected to constant temperature is given in Table 2. It observed from this table that as the distance from the support is varied there is no substantial change in stresses. It is also observed from this table that the stresses calculated by the analytical solution agree very closely with FE results in the region away from the ends of the strip.

### **Stresses in Bi-material Strip Subjected to Axial Temperature Gradient and its Comparison with Bi-material Strip Subjected to Constant Temperature**

The stresses in bi-material strip subjected to constant temperature are already discussed. The bi-material strip is subjected to axial temperature gradient and analysis is carried out. The results are tabulated in Table 3. From the table it is observed that stresses are maximum at support, Further it is seen that stress distribution shows stresses increases along length towards the end having maximum temperature. The bi-material strip subjected to constant temperature has maximum temperature at support as in case of bi-material strip subjected to axial temperature gradient. Moreover in bi-material strip subjected to constant temperature has no significant increase in stresses along length while in bi-material strip subjected to axial temperature gradient the stresses along length increases towards the end having maximum temperature.

## **CONCLUSION**

From this study it is concluded that when the thickness of both strips of bi-material is same then analytical and FE stresses are constant throughout the span at constant temperature loading but FE stress is maximum at the support. Hence analytical solution only gives the stresses away from the support. When such strip is subjected to axial temperature gradient then stresses varies along the length according to temperature and maximum at support although temperature is minimum.

## **REFERENCES**

1. Aleck B J (1949), "Thermal Stresses in a Rectangular Plate Clamped along an Edge", *ASME Journal of Applied Mechanics*, Vol. 16, pp. 118-122.
2. Born J S and Horvay G (1955), "Thermal Stresses in Rectangular Strips – II", *Journal of Applied Mechanics*, Vol. 22, pp. 401-406.
3. Durelli A J and Tsao C H (1955), "Determination of Thermal Stresses in Three-Ply Laminates", *ASME Journal of Applied Mechanics*, Vol. 77, pp. 190-192.
4. Eischen J W and Everett J S (1989), "Thermal Stress Analysis of a Bi-material Strip Subject to an Axial Temperature Gradient", *ASME Journal of Electronic Packaging*, Vol. 111, pp. 282-287.
5. Eischen J W, Chung C and Kim J H (1990), "Realistic Modeling of Edge Effect Stresses in Bi-material Elements", *ASME Journal of Electronic Packaging*, Vol. 112, pp. 16-22.
6. Khurmi R S and Gupta J K (1999), "A Textbook of Machine Design", EPH Publication.

7. Rao Venkateshwara A, Prasad K S V, Avinash M, Nagababu K, Manohar V, Raju P S R and Chandra G R (2012), "A Study on Deflection of a Bi-metallic Beam Under Thermal Loading Using Finite Element Analysis", *IJEAT Journal*, Vol. 2, pp. 81-82.
8. Sathe Madhulika (2004), "Design, Fabrication and Thermomechanical Testing of a Vertical Bimorph Sensor in the Wafer Plane", A Thesis Submitted to Louisiana State University and Agriculture and Mechanical College, pp. 6-44.
9. Suhir E (1986), "Stresses in Bi-metal Thermostats", *ASME Journal of Applied Mechanics*, Vol. 53, pp. 657-660.
10. Suhir E (1989), "Interfacial Stresses in Bi-metal Thermostats", *ASME Journal of Applied Mechanics*, Vol. 56, No. 3, pp. 595-600.
11. Timoshenko Stephen P and Gere James (2010), "Buckling of a Bi-metallic Strip", *Theory of Elastic Stability*, 2<sup>nd</sup> Edition, pp. 310-313.