

Research Paper

# FREE VIBRATION ANALYSIS OF THIN FRP SKEW SYMMETRIC LAMINATE WITH CIRCULAR CUT-OUT

U Anitha<sup>1\*</sup>, P Hussain<sup>1</sup>, A Ramanjaneya Reddy<sup>1</sup> and S Sudhakar Babu<sup>1</sup>

\*Corresponding Author: **U Anitha** ✉ [uanitha.nth@gmail.com](mailto:uanitha.nth@gmail.com)

The present analysis deals with the prediction of natural frequencies of a skew laminated FRP plate with circular cut-out. The scope of the work includes investigation of the influence of skew angle, size of the hole, length to thickness ratio, fiber angle orientation on natural frequencies. The length-to-thickness ratio is so chosen that the plate can be considered as thin plate, which can be analyzed using classical lamination theory with reasonable accuracy. Four-layered symmetric cross-ply ( $0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ}$ ) and angle-ply ( $\theta^{\circ}/-\theta^{\circ}/-\theta^{\circ}/\theta^{\circ}$ ) laminates are considered for the analysis. The first five natural frequencies are obtained by varying the parameters skew angle ( $\alpha$ ), diameter of the hole to length of the plate ratio ( $d/l$ ), length to thickness ratio ( $s$ ) and fiber orientation ( $\theta$ ) by taking the length of the sides of the skew plate as 1 m. The problem is modeled with layered element of ANSYS software which is designed based on Classical Lamination Theory (CLT).

**Keywords:** Pavement failure, Bituminous Roads, Traffic Study, Pavement Design, IRC 37: 2001

## INTRODUCTION

In the recent period, FRP composites have many benefits to their selection and use. The selection of the materials depends on the performance and intended use of the product. The composites designer can tailor the performance of the end product with proper selection of materials. It is important for the end-user to understand the application environment, load performance and durability requirements of the product and convey this information to the composites industry professional. The low density, high strength-to-weight ratio, directional strength, corrosion

resistance, weather resistance, dimensional stability, excellent durability and design flexibility of fiber-reinforced composites are the primary reasons for their use in many structural components, in aircraft, automotive, marine and other industries.

Fiber-reinforced composites are now used in applications ranging from space craft frames to ladder rails, from aircraft wings to automobile doors, from rocket motor cases to oxygen tanks and from printed circuit boards to tennis rackets. Their use is increasing at such a rapid rate that they are no longer considered advanced materials.

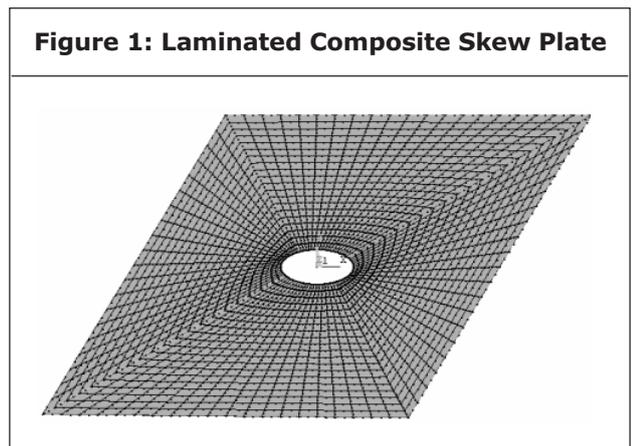
<sup>1</sup> Department of Mechanical Engineering, AVR&SVR Engineering College, nannur(v), Kurnool(Dt), India.

The essence of fiber-reinforced composite technology is the ability to put strong stiff fibers in the right place in the right orientation and right volume fraction.

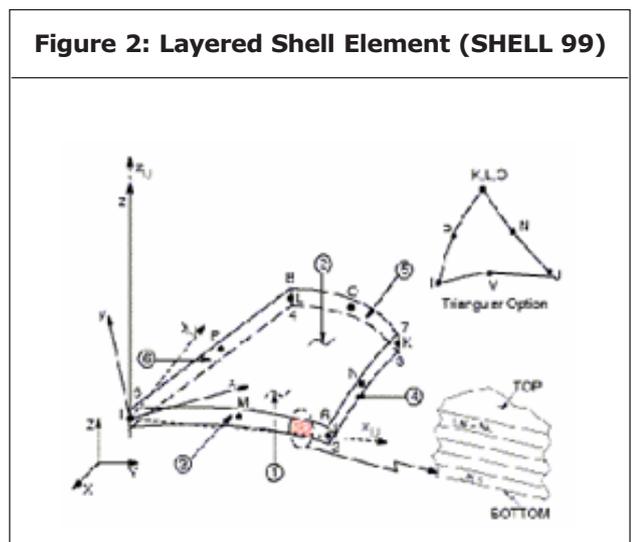
**Modeling the Problem**

**Geometric Modeling**

The geometry of the problem is shown in Figure 1. The sides of the plate 'l' are taken equal to 1 m. The laminate considered for the present analysis is a four-layered symmetric laminate. The skew angle is varied from 0° to 50° and the major axis of the circular cut-out is selected from the a/l ratio which is varied from 0.2 to 0.6, the number of layers are varied as 4, 8, 16, 24 and the length-to-thickness ratio is ranged from 20-100.



**Figure 1: Laminated Composite Skew Plate**



**Figure 2: Layered Shell Element (SHELL 99)**

**Finite Element Modeling**

Laminated composite general shell element SHELL99 of ANSYS is used for meshing the geometry of the problem. This element is suited for modeling moderately thick to thin laminated composite shells. As shown in Figure 2, the element consists of number of layers of perfectly bonded orthotropic materials.

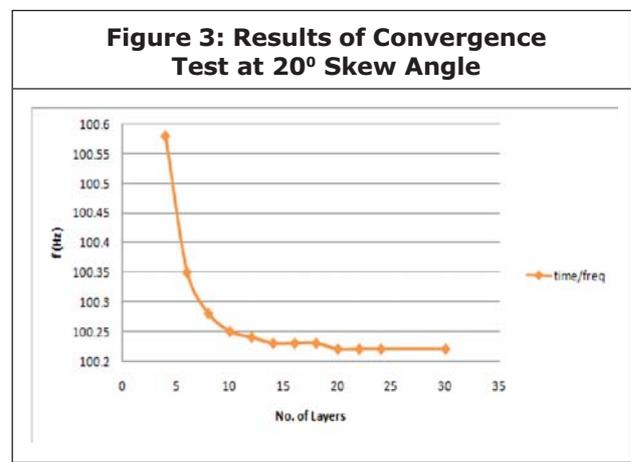
**RESULTS AND DISCUSSION**

**Analysis of Results**

In this chapter, the convergence of the finite element model and the results obtained for the first five natural frequencies of the four-layered symmetric thin skew FRP laminate with circular cut-out by varying parameters i) Skew angle ( $\alpha$ ); (ii) Number of layers (n); (iii) Half the major axis of hole to length of skew plate ratio (a/l); (iv) length to thickness ratio (s); (v) fiber orientation ( $\theta$ ) are presented for cross-ply and angle-ply laminates respectively and the effect of each parameter on the natural frequencies is discussed.

**Convergence Test**

The density of the finite element mesh is varied by increasing the number of divisions along the edges of the plate and the variation of natural frequency with respect to the number of elements on the edges is plotted in Figure 3. It can be



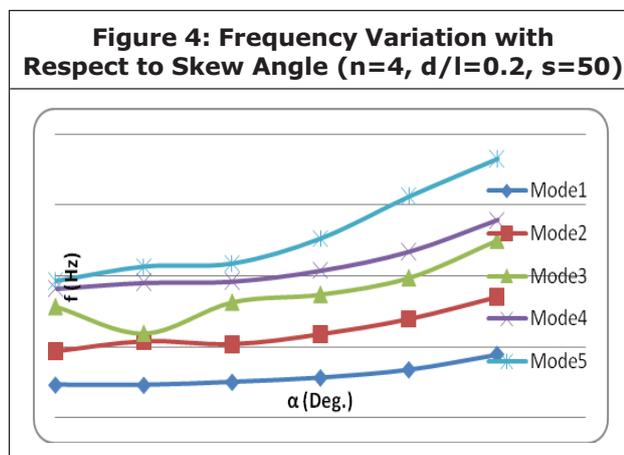
**Figure 3: Results of Convergence Test at 20° Skew Angle**

observed that the results are converged for number of element divisions above 18 and thus results are taken at 20 divisions.

### PART-I (FOR CROSS-PLY LAMINATES)

#### Effect of skew angle

The effect of skew angle on the fundamental frequency of vibration for the first five modes is studied by analyzing four layered cross-ply ( $0^\circ/90^\circ/90^\circ/0^\circ$ ) thin Carbon-epoxy laminates with 's'=50 and d/l ratio=0.2 and all edges simply supported by keeping the value of 'l' constant as 1m in all cases. By variation of skew angle ( $\alpha$ ) from  $[0^\circ-50^\circ]$ , the variation in natural frequency is observed. It has been observed that the natural frequency is increasing with increase in the skew angle (Figure 4). This is due to the increase in resultant stiffness and decrease in mass of the skew plate with increase in skew angle. The rate of increase is different for different modes, which are due to the specific deformed mode of the plate. Figure 5 shows the variation of the natural frequency with respect to mode number for a skew angle of  $0^\circ$ . Here in each mode there is a variation in principal curvatures of the deformed plate that results in variation of the structural rigidity which in turn varies the natural frequency of the



structure. We can interpret the similar reasons for other skew angles considered.

### PART-II (FOR ANGLE-PLY LAMINATES)

#### Effect of Fiber/LAYER ORIENTATION on Skew Angle

The effect of fiber orientation on the fundamental frequency of vibration for the first five modes is studied by analyzing four layered thin Carbon-epoxy laminates with 'l'=1 and all edges clamped. With increase in fiber angle from  $0^\circ$  to  $90^\circ$  frequencies are found to have increase and decrease pattern and it is different for different modes as shown in Figures 5 to 12.

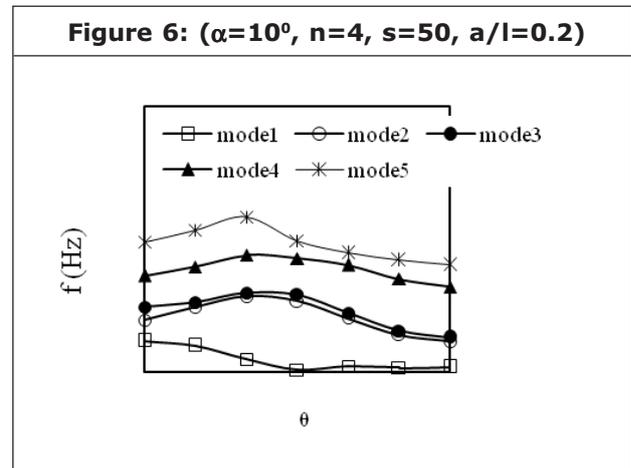
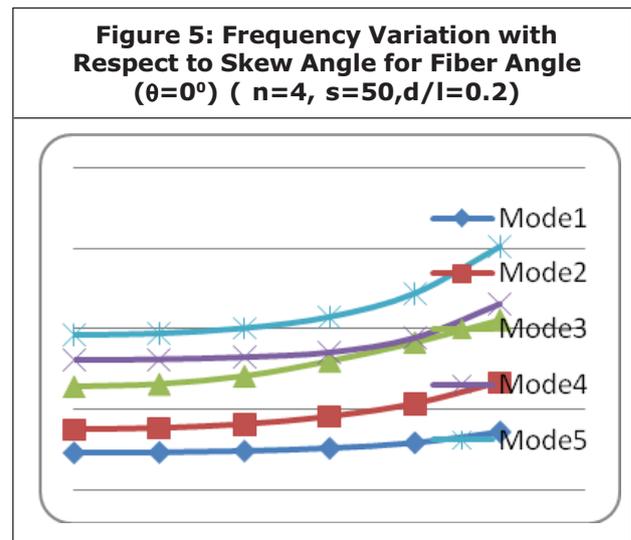


Figure 7: ( $\alpha=20^\circ, n=4, s=50, a/l=0.2$ )

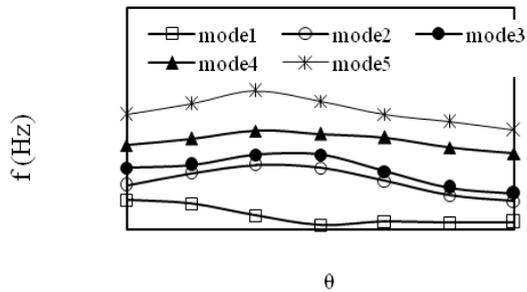


Figure 8: ( $\alpha=30^\circ, n=4, s=50, a/l=0.2$ )

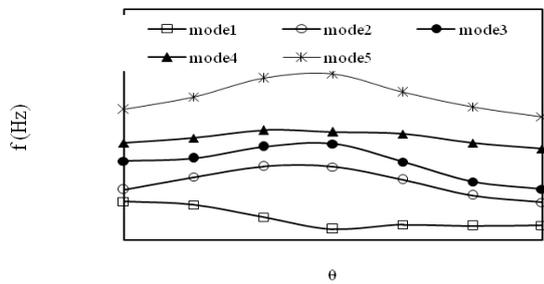


Figure 9: ( $\alpha=40^\circ, n=4, s=50, a/l=0.2$ )

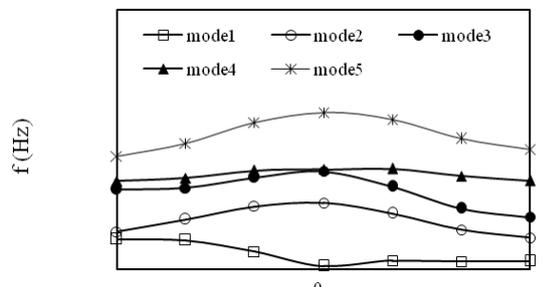


Figure 10: ( $\alpha=50^\circ, n=4, s=50, a/l=0.2$ )

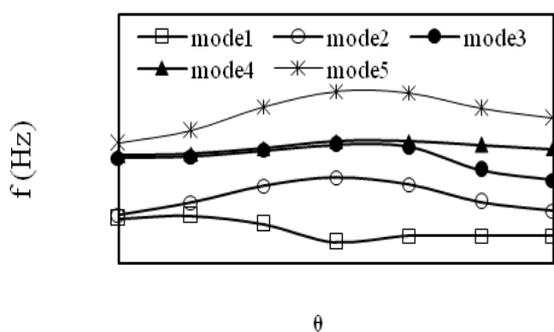


Figure 11: ( $\alpha=60^\circ, n=4, s=50, a/l=0.2$ )

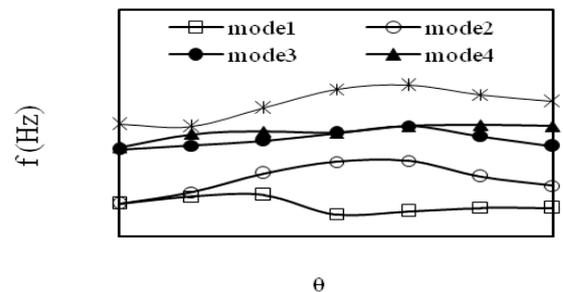
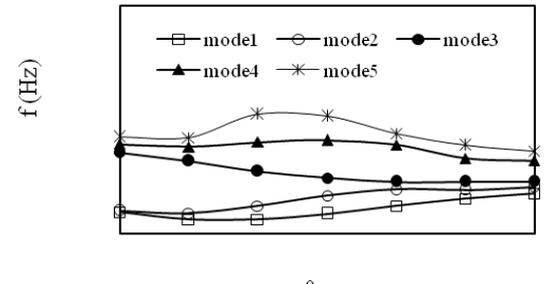


Figure 12:  $a/l=0.2(n=4, s=50, \alpha=60^\circ)$



## CONCLUSION

- Natural frequency increases with increase in skew angle from  $(0^\circ-60^\circ)$  and is different for different mode shape. As skew angle increases the internal stiffness of the skew plate increases and the mass of the plate remains constant. As a result the natural frequency increases.
- Natural frequency increases up to  $n=8$  and later there is no significant change with respect to 'n'.
- Natural frequency increases with increase in half the major axis of hole to length of the plate ratio ( $a/l$ ) from 0.1-0.6. As  $a/l$  ratio increases the internal stiffness of the skew plate increases and the mass of the plate decreases. As a result the natural frequency increases and variation of increase is different for different modes.

- Natural frequency increases with increase in thickness to length ratio ( $s$ ) from (20-100) of skew plate. As thickness increases both stiffness and mass of the skew plate contribute to increase natural frequency at a greater rate.
- Natural frequency varies with increase in fiber angle ( $\theta$ ) from ( $0^\circ$ - $90^\circ$ ) for all skew angles. With the increase in stiffness of the skew plate, fiber direction of the laminate may change which in turn greatly effects the variation in natural frequency and is different for different modes.
- Natural frequency varies with increase in fiber angle ( $\theta$ ) from ( $0^\circ$ - $90^\circ$ ) for  $a/l$  ratio (0.1-0.6). With the increase in internal stiffness and mass contribution fiber angle may change which in turn shows effect on natural frequency.

## REFERENCES

1. Andrew S Bicos and George S Springer (1989), "Analysis of free damped vibration of laminated composite plates and shells", *International Journal of Solids and Structures*, Vol. 25, pp. 129-149.
2. Ju F, Lee H P and Lee K H (1995), "Free vibration of composite plates with delaminations around cutouts", *Composite structures*, Vol. 31, pp 177-183.
3. Lam K Y, Hung K C and Chow S T (1989), "Vibration analysis of plates with cutouts by the modified Rayleigh-Ritz method", *Journal of Applied Acoustics*, Vol. 28, pp. 49-60.
4. Payman Afshari G E O Widera (2000), "Free vibration analysis of composite plates", *Journal of Pressure vessel technologies*, Vol. 122, pp. 390-399.
5. Prabhakaran R and Rajamani A (1978), "Free vibration characteristics of clamped-clamped composite plates with circular holes and square cut-outs", *Conf. Composite materials*, pp. 367-375.
6. Ramakrishna S, Koganti M Rao and Rao N S (1992), "Free vibration analysis of laminates with circular cutout by hybrid stress finite element", *Journal of Composite structures*, Vol.21, pp. 177-185.