International Journal of Engineering Research and Science & Technology

IJERST

ISSN : 2319-5991
Vol. 4, No. 3
August 2015

www.ijerst.com

Email: editorijerst@gmail.com or editor@ijerst.com

International Journal of Engineering Research and Science & Technology

ISSN 2319-5991 www.ijerst.com
Vol. 4, No. 3, August 2015

Research Paper

MINIMIZING RENTAL COST IN 3-MACHINE SPECIALLY STRUCTURED FLOWSHOP PROBLEMS

Laxmi Narain[1]*

**Corresponding Author: Laxmi Narain* ✉ laxminarain@andc.du.ac.in*

This paper studies 3-machine specially structured flow-shop problems under rental *Policy III*, where processing times of jobs on various machines follow certain conditions. *Policy III*, it is considered that first machine will be taken on rent in the starting of processing the jobs; second machine will be taken on rent when the first job is completed on first machine and third machine is taken on rent when first job is completed on second machine. The objective is to obtain a sequence which minimizes the total rental cost of the machines. A simple algorithm is developed without using Branch-and-Bound technique. The algorithm is illustrated through a numerical example.

Keywords: Flow-shop Scheduling, Idle Time, Elapsed Time, Rental Time, Rental Cost

INTRODUCTION

In flow-shop sequencing problem, when one has got the assignment but does not have one's own machines or does not have enough money or does not want to take risk of investing money for the purchase of machines, under these circumstances, may take machines on rent to complete the assignment. Minimization of total rental cost of machines will be the criterion in these types of situations.

The following renting policies generally exist:

Policy I: All the machines are taken on rent at one time and are returned also at one time.

Policy II: All the machines are taken on rent at one time and are returned as and when they are no longer required.

Policy III: All the machines are taken on rent as and when they are required and are returned as and when they are no longer required for processing.

Various authors (Bagga, 1969; Bagga and Ambika Bhambani, 1996; Bagga and Khurana, 1983; Narain and Bagga, 2004; Narain and Bagga, 2005; Panwalker and Khan, 1975; Rinnooy Kan *et al.*, 1975) studied these rental policies to optimize the given objective function. Under *Policy I*; the sequence which minimizes the total elapsed time will also minimizes the total rental cost of the machines. For 2-machine problem Johnson

[1] Associate Professor, Department of Mathematics, Acharya Narendra Dev College, University of Delhi, Delhi, India.

(1954); for 3-machine problem Lomnicki (1965) and Bagga (1969); and for m-machine problem Gupta (1970) and Panwalker and Khan (1975) can be applied to minimize the total rental cost of machines. Under *Policy II*; in case of 2-machine flow-shop problem, Johnson (1954) will provide the optimal sequence. For 3-machine flow-shop problem, Bagga and Ambika (1996) provided a Branch-and-Bound algorithm.Bagga and Khurana (1983) use Branch-and-Bound technique to solve n-job, 2-machine flow-shop problems under two policies: (i) when both machines are hired simultaneously and (ii) when the 2nd machine is hired only when the first job is completed on 1st machine.

In this Paper, *PolicyIII* is adopted for 3-machine specially structured flow-shop problems. For three-machine specially structured flow-shop problems, the restrictions on the processing times of the jobs considered by Johnson (1954) are being relaxed. The relaxed restriction on the processing times is any of the following:

- Processing time of any job on the first machine is never less than the processing time of any of remaining jobs on the middle machine.
- Processing time of any job on the last machine is never less than the processing time of any of the remaining jobs on the middle machine.

A simple algorithm, without using Branch-and-Bound technique, has been obtained for these cases.Numerical example is given to demonstrate the algorithm.

MATHEMATICAL FORMULATION

Notations

S : Sequence of jobs 1,2,…,n.

M_j : Machine j, j = 1, 2, 3.

p_{ij} : Processing time for job i on machine M_j.

I_{ij} : Idle time of machineM_j for job i.

C_j : Rental cost per unit time of machine M_j, j = 1, 2, 3.

J_r : Partial schedule of r scheduled jobs.

J_r' : Set of remaining (n-r) free jobs.

i_1 : Job at 1st position of partial schedule J_r.

$Z_{ij}(S)$: Completion time of ith job of sequence S on machine M_j.

$Z_{i1,j}(S)$: Completion time of job i_1 of sequence S on machine M_j.

t(Jr, j) : Time when the last job of the assigned schedule J_r is completed on machine M_j, j = 1, 2, 3.

LB[Jr] : Lowest possible rental cost corresponding to partial schedule J_r, irrespective of any schedule of J_r'.

Let n jobs require processing over three machines M_1, M_2 and M_3 in the order $M_1 \rightarrow M_2 \rightarrow M_3$.

Without any loss of generality, we can assume that the jobs are processed according to sequence S, where S = 1, 2,…, n.

For sequence S, total elapsed time

$$T(S) = \sum_{i=1}^{n} p_{i3} + \sum_{i=1}^{n} I_{i3}$$

$$= \sum_{i=1}^{n} p_{i3} + \max_{1\le u\le v\le n} \left(H_v + K_u\right) \quad ...(2.1)$$

where $$H_v = \sum_{i=1}^{v} p_{i2} - \sum_{i=1}^{v-1} p_{i3} \quad ...\ (2.2)$$

$$K_u = \sum_{i=1}^{u} p_{i1} - \sum_{i=1}^{u-1} p_{i2} \quad ...\ (2.3)$$

Case 1: $p_{j2} \le p_{i1}$; $\forall$i & j; i $\ne$ j

i.e.,

$$\max_j \ p_{j2} \le \min_i \ p_{i1}; \ \forall \ i, \ j; \ i \ne j$$

From equation (2.3);

$$K_4 = \sum_{i=1}^{4} p_{i1} - \sum_{i=1}^{4-1} p_{i2}$$

Here,

$K_1 = p_{11}$

$K_2 = p_{11} + p_{21} – p_{12}$

$= p_{11} + (p_{21} – p_{12})$

$\ge p_{11}$ (since $p_{21} \ge p_{21}$)

Therefore, $K_2 \ge K_1$

$K_3 = p_{11} + p_{21} + p_{31} – p_{12} – p_{22}$

$= p_{11} + p_{21} – p_{12} + (p_{31} – p_{22})$

$= K_2 + (p_{31} – p_{22})$

$\ge K_2$ (since $p_{31} \ge p_{22}$)

Therefore, $K_3^3 K_2^3 K_1$

Continuing in this way;

$K_n \ge K_{n-1} \ge \ldots \ge K_1$

i.e., $K_1 \le K_2 \le \ldots \le K_n$

i.e., $K_u \le K_{u+1} \le \ldots \le K_v$, where $1 \le u \le v \le n$

i.e., sequence $\{K_u\}$ is monotonically increasing.

Thus $K_u \le K_v$ when $u \le v$

Therefore, on replacing K_u by K_v,

$$\sum_{i=1}^{n} I_{i3} = \max_{1 \le v \le n} \ (H_v + K_v)$$

$$= \max_{1 \le v \le n} \left[\sum_{i=1}^{v} (p_{i1} + p_{i2}) - \sum_{i=1}^{v-1} (p_{i2} + p_{i3}) \right] \quad \ldots (2.4)$$

Case 2: $p_{i2} \le p_{j3}$; $\forall$i& j; i $\ne$ j

i.e.,

$$\max_i \ p_{i2} \le \min_j \ p_{j3}; \ \forall \ i, \ j \ ; \ i \ne j$$

From equation (2.2);

$$H_v = \sum_{i=1}^{v} p_{i2} - \sum_{i=1}^{v-1} p_{i3}$$

$H_1 = p_{12}$

$H_2 = p_{12} + p_{22} – p_{13}$

$= p_{12} + (p_{22} – p_{13})$

$\le p_{12}$ (since $p_{22} \le p_{23}$)

Therefore, $H_2 \le H_1$

$H_3 = p_{12} + p_{22} + p_{32} – p_{13} – p_{23}$

$= p_{12} + p_{22} – p_{13} + (p_{32} – p_{23})$

$= H_2 + (p_{32} – p_{23})$

$\le H_2$ (since $p_{32} \le p_{23}$)

Therefore, $H_3 \le H_2 \le H_1$

i.e., $H_1 \ge H_2 \ge H_3$

Continuing in this way;

$H_1 \ge H_2 \ge_3 \ldots \ge H_n$

i.e., $H_n \le H_{n-1} \le \ldots \le H_1$

i.e.,$H_v \le H_{v-1} \le \ldots \le H_u$

i.e., sequence $\{H_v\}$ is monotonically decreasing.

Therefore, = $=\max_{1 \le u \le v \le n} (H_u + K_v)$

$$= \max_{1 \le u \le n} \left(H_u + K_4\right)$$

$$= \max_{1 \le u \le n} \left(\sum_{i=1}^{u} (p_{i1} + p_{i2}) - \sum_{i=1}^{u-1} (p_{i2} + p_{i3}) \right)$$

… (2.5)

From equations (2.4) and (2.5), it is clear that three-machine flow-shop problem is solvable by converting it into two-machine flow-shop problem. Under *Policy III*, the following algorithm will provide the optimal solution for both Case 1 and Case 2 of three-machine specially structured flow-shop problems.

ALGORITHM:

Step 1: Compute

$G_i = p_{i1} + p_{i2}$

$H_i = p_{i2} + p_{i3}$; i = 1, 2, …, n

Step 2: Obtain the sequence S_1 (say) by applying Johnson's two-machine algorithm on machine G and H.

Step 3: Obtain other sequences by putting 2^{nd}, 3^{rd}… n^{th} job of sequence S_1 in the first position and other jobs of S_1 in the same order. Let these sequences be S_2, S_3, …, S_n respectively.

Step 4: For sequences S_1, S_2, …, S_n ; compute

$R(S\text{-}_{i\text{-}}) = [Z_{n2} (S_i) – Z_{11} (S_i)] \times C_2 + [Z_{n3} (S_i) – Z_{12} (S_{i\text{-}})] \times C_3$;

i = 1, 2, …, n

Step 5: Compute the minimum of $R(S_1)$, $R(S_2)$,…, $R(S_n)$. Let this minimum be for sequence S_r. Then S_r is an optimal sequence and for this sequence calculate total rental cost of the machines.

EXAMPLE

Example 4.1: Consider a 4-Job, 3-Machine sequencing problem with processing times (in hours) as given in Table 1. Rental cost of M_1, M_2 and M_3 are 4 units, 7 units and 5 units respectively.

Here, $p_{i2} \leq p_{j3}$; $\forall$i& j; i ≠ j

Applying algorithm 3.1;

Step 1: The processing times of 4 jobs on 2 fictious machines G and H are given as in Table 2.

Step 2: The sequences generated by applying Johnson's two-machine algorithm on machines G and H is S_1 = 2-4-1-3.

Step 3: The sequences generated by putting the 2^{nd}, 3^{rd} and 4^{th} job of sequence S_1 in the first position and other jobs of the sequence S_1 in the same order are S_2 = 4-2-1-3; S_3 = 1-2-4-3 and S_4 = 3-2-4-1.

Step 4: For determining the minimum total rental cost, these four sequences are enumerated and their completion time In-Out tables are shown as in Table 3; Table 4; Table 5 and Table 6.

For Sequence S_1 = 2-4-1-3;

Total rental cost on M_2 and M_3

= (21-3) × 7 + (33-5) × 5

= 18 × 7 + 28 × 5

=126 + 140 = 266 units.

For Sequence S_2 = 4-2-1-3;

Total rental cost on M_2 and M_3

= (21-2) × 7 + (34-6) × 5

= 19 × 7 + 28 × 5

= 133 + 140 = 273 units.

For Sequence S_3 = 1-2-4-3;

Total rental cost on M_2 and M_3

= (21-5) × 7 + (34-6) × 5

= 16 × 7 + 28 × 5

= 112 + 140 = 252 units

Table 1: Processing Times of Jobs on Machines

Jobs	Machines		
	M_1	M_2	M_3
1	5	1	8
2	3	2	9
3	6	5	5
4	2	4	6

Table 2: Processing Times of Jobs on Fictious Machines

Jobs	Machines	
	G	H
1	6	9
2	5	11
3	11	10
4	6	10

Table 3: Completion Time in-Out of Sequence S_1

Jobs	Machines		
	M_1 In-Out	M_2 In-Out	M_3 In-Out
2	0-3	3-5	5-14
4	3-5	5-9	14-20
1	5-10	10-11	20-28
3	10-16	16-21	28-33

Table 4: Completion Time in-Out of Sequence S_2

Jobs	Machines		
	M_1 In-Out	M_2 In-Out	M_3 In-Out
4	0-2	2-6	6-12
2	2-5	6-8	12-21
1	5-10	10-11	21-29
3	10-16	16-21	29-34

Table 5: Completion Time in-Out of Sequence S_3

Jobs	Machines		
	M_1 In-Out	M_2 In-Out	M_3 In-Out
1	0-5	5-6	6-14
2	5-8	8-10	14-23
4	8-10	10-14	23-29
3	10-16	16-21	29-34

Table 6: Completion Time in-Out of Sequence S_4

Jobs	Machines		
	M_1 In-Out	M_2 In-Out	M_3 In-Out
3	0-6	6-11	11-16
2	6-9	11-13	16-25
4	9-11	13-17	25-31
1	11-16	17-18	31-39

For Sequence S_4 = 3-2-4-1;

Total rental cost on M_2 and M_3

$$= (18-6) \times 7 + (39-11) \times 5$$

$$= 12 \times 7 + 28 \times 5$$

$$= 84 + 140 = 224 \text{ units.}$$

Step 5: Total rental cost on machines M_2 and M_3 for sequences S_1, S_2, S_3 and S_4 are 266; 273; 252 and 224 units respectively. Minimum of {266, 273, 252, 224} = 224 units. This minimum value is corresponding to the sequence 3-2-4-1. Hence, the optimal sequence is 3-2-4-1 and total rental cost = 16 × 4 + 224 = 288 units.

CONCLUSION

In this paper, 3-machine specially structured flow-shop problem is considered under rental *Policy III*. Here it is proved that this three-machine problem can be converted into two-machine problem. We have given a very simple and efficient algorithm without using branch and bound technique.

REFERENCES

1. Bagga P C (1969), “Sequencing in a Rental Situation”, Jr. of Canadian Operations Research Society, Vol. 7, pp. 152-153.
2. Bagga P C and Ambika Bhambani (1996), “Minimizing Rental Costs in Three-machine Sequencing Problem”, *Jr. of Indian Association for Productivity, Quality and Reliability,* Vol. 21, pp. 73-77.
3. Bagga P C and Khurana K (1983), “Minimizing Waiting Cost of Jobs in Renting Situations”, *Journal of Indian Association for Production, Quality and Reliabilit*y, Vol. 8, No. 1.
4. Gupta J N D (1970), “*m*-Satge Flow-shop Scheduling by Branch and Bound”, *OPSEARCH,* Vol. 7, pp. 37.
5. Johnson S M (1954), “Optimal Two and Three Stage Production Schedule with Set up Times Included”, *Naval Research Logistics Quarterly*, Vol. 1, pp. 61-68.
6. Lomnicki Z A (1965), “A Branch-and-Bound Algorithm for the Exact Solution of the Three-machine Scheduling Problem”, *Operational Research Quarterly,* Vol. 16, pp. 89-100.
7. Narain L and Bagga P C (2004), “Flow-shop/ No-idle Scheduling to Minimize Total Elapsed Time”, *Journal of Global Optimization*, Vol. 33, pp. 349-367.
8. Narain L and Bagga P C (2005), “Flow-shop/ No-idle Scheduling to Minimize Mean Flow-time”, *ANZIAM. Journal,* Vol. 47, pp. 265-275.
9. Panwalker S S and Khan A W (1975), “An Improved Branch and Bound Procedure for the *nxm* Flow-shop Problems”, *Naval Research Logistics Quarterly,* Vol. 22, pp. 787-790.
10. Rinnooy Kan A H G, Lageweg B G and Lenstra J K (1975), “Minimizing Total Costs in One Machine Scheduling”, *Operations Research,* Vol. 23, pp. 908-927.

International Journal of Engineering Research and Science & Technology
Hyderabad, INDIA. Ph: +91-09441351700, 09059645577
E-mail: editorijlerst@gmail.com or editor@ijerst.com
Website: www.ijerst.com

ISSN 2319-5991