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Research Paper

HASHIN'S FAILURE CRITERIA IN-PLANE STRESS NUMERICAL MODEL CORRELATED TO TENSILE TESTS

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3D failure models to predict damage in composite structures tends to use much more CPU time than 2D models due to its complexity and greater element and node quantity. This paper will present a good correlation between experimental and numerical using the ABAQUS software with its implemented Hashin's Failure Criteria in a 2D model using less computational resources. Tabbed test coupons were subjected to uniaxial loading for the experimental part in order to check and correlate results given in the Finite Element Analysis (FEA).

Keywords: Carbon-epoxy Composite, FEA, ABAQUS, Hashin, Tabbed specimen, Tensile test

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INTRODUCTION

Many structures used these days need a different approach regarding materials, since metal, ceramics and polymers do not have the desired characteristics. Structures used in automobile and aeroespacial industries need high strength, high modulus (elastic modulus) and low density, and sometimes chemical resistance also, which cannot be achieved using a singular material.

Structures must maintain their initial dimensions under load and that's the main reason for high modulus materials be desired. Metals have high modulus (> 50 GPa), however their density tends to be high too. Polymers have low density but they lack high elastic modulus.

METHODS

In this study, damage to a composite coupon was verified in a Finite Element Analysis (FEA) model. Damage in composite structures is a complex phenomenon that can happen through various failure mechanisms like fiber tension damage, fiber compression (buckling), matrix tension and compression damage, fiber and matrix debonding and/or delamination. Computational resources have increased in the past decade but so did the complexity and size of composite structure along with different criteria formulations. To get a reliable prediction of such phenomenon with a certain degree of accuracy is necessary to implement the correct finite element method calculation. Indeed, it needs more studies like this

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to better understand the mechanics of failure to improve numerical failure models for composite materials.

The damage models in composites can be divided into four different modes:

1. Failure criteria
2. Fracture mechanics
3. Plasticity
4. Damage mechanics

Failure criteria modes were initially developed for unidirectional laminate materials and only in static regime. They are divided into two categories: interactive and Noninteractive criteria according to Jones (1999). Noninteractive criteria have decoupled failure modes and specific expressions are used to identify failure mechanisms. In a stress based criteria, every stress in the principal coordinates must be less than the respective strengths or fracture will occur. The same for maximum strain failure criteria, the failure occurs if one of the strains in the principal coordinates exceeds its failure strain.

Naturally, for any material there is a relation between stress and strain as follows:

ε = or σ

These equations specify a constitutive law, and for that they are called constitutive equations. They introduce a concept to model ideal materials to real problems.

Differently, interactive criteria use an interaction between two or more failure mechanisms and then describe the failure surface for strain or stress domains. Commonly, stress/strain polynomial expressions describe the boundaries for failure envelope. Any stress/strain inside the envelope means no failure for the structure. Interactive failure criteria are easily found in the open literature as the Tsai and Wu (1971) criterion, Tsai-Hill criterion, Hoffman criteria and others. On the other hand there is a disadvantage in using polynomial criteria, since they don't say anything about damage mechanisms and therefore modified versions are used to differ failure modes. Hashin (1980) proposed three-dimensional failure criteria for unidirectional composites in plane stress.

In continuum mechanics, a material is said to be under plane stress if the stress vector is zero across a particular surface. When that situation occurs over an entire element of a structure, as is often the case for thin plates, the stress analysis is considerably simplified, as the stress state can be represented by a tensor of dimension 2 rather than a 3x3 matrix.

His model shows four different failure modes (fiber failure in tension or compression and matrix buckling in tension or cracking in compression) that are modelled separately.

Engblom and Havelka (1989) have proposed a combination of the Hashin and Lee failure criteria. Hashin's criterion was used to detect in-plane failure and the Lee's criteria predicted delamination, and reducing the stresses associated to each failure mechanisms to zero in the composite material constitutive law. This combination won't be used in this study since the experimental part only considers tensile tests and does not consider compressive tests.

HASHIN CRITERIA

The quadratic form used by Tsai Wu criteria is limited to represent many failure modes simultaneously, due to the inexistence of many modes, and does not indicate directly the failure cause. To determine the normal stress interaction parameters is difficult due to minor experimental measuring errors produce a big difference on the failure envelope. Hashin has proposed a failure criteria divided in sub-criteria capable of determining each failure mode by smooth forms. This criteria is tridimensional for composites with unidirectional fiber and transversely isotropic using a quadratic stress polynomial. Hashin's criteria does not solve all possible failure modes, but it address the four main failure modes: tensile and compression of the fibers and tensile and compression of the matrix. The F_{12} parameter that is difficult to obtain from the Tsai Wu method isn't needed for Hashin's criteria, since the failure modes are independent of σ_2 tension and the matrix failure modes are independent of σ_1.

The damage to the material is considered anisotropic, but the behavior pre damage is linearly elastic. This model is in plane stress state and therefore shell elements were used. If tridimensional elements were needed, a subroutine coded in python must be written in order to ABAQUS software use Hashin's criteria. Post damage response is equalized by the crack propagation model. The failure modes in plane stress state are the following:

- Mode 1: Fiber Tension (σ_{11}> 0)

$$f_I = \left(\frac{\hat{\sigma}_{11}}{X^T}\right)^2 + \alpha \text{, where } 0 <$$

- Mode 2: Fiber Compression (σ_{11}> 0)

$$f_{II} = \left(\frac{\hat{\sigma}_{11}}{X^C}\right)^2$$

- Mode 3: Matrix Tension (σ_{22}> 0)

$$f_{III} = \left(\frac{\hat{\sigma}_{22}}{Y^T}\right)^2 + \left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2$$

- Mode 4: Matrix compression (σ_{22}< 0)

$$f_{IV} = \left(\frac{\hat{\sigma}_{22}}{2S^T}\right)^2 + \left[\left(\frac{Y_C}{2S^T}\right) - 1\right]\frac{\hat{\sigma}_{22}}{Y^C}\left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2$$

Matrix compression failure includes the admissible transverse shearing stress S^T, which is not acceptable from a physics point of view, so this failure criterion does not involve a stress component but involves it admissible value.

σ_{11}, σ_{22} are effective stress tensor components, which are used to evaluate the crack initiation criterion and follows the formula:

$$\hat{\sigma} = M\sigma$$

where σ is the real stress and is the failure operator as follows:

$$M = \begin{pmatrix} \frac{1}{(1-d_f)} & 0 & 0 \\ 0 & \frac{1}{(1-d_m)} & 0 \\ 0 & 0 & \frac{1}{(1-d_s)} \end{pmatrix}$$

The historical significance of these criteria is due to the conception of different failure criterion for composite materials. Hashin started identifying the prevailing failure modes, and then the variables related to these modes. Afterwards he proposed interactions between variables of each mode (ParÍs, 2001). As said before, the recognition of the failure mode is important so progressive failure analysis (like LPF) can be used in structural components made by composite materials. The mathematical procedure starts analyzing stresses by finite element analysis. Some elements might fail when stresses match the failure criterion, but the continuity of the analysis is essential to determine which failure mode was satisfied. With this information, new laminate may be assembled in order to modify the structural strength in that particular area, and then go on with the analysis.

MATERIALS (SPECIMEN)

In order to acquire data by the tensile testing, the specimen needs to be manufactured with tabs. For this study the woven used were biaxial given the laminate the same properties in each of the main direction, but not quasi-isotropic. The tabbed specimen were manufactured according to ASTM D3039, 2008 standard.

There are two reasons for tabbed specimen: protect the surface that is in contact with the grip which applies a huge compression force and usually damages the specimen, and the other reason is for increasing the contact area distributing the loads more uniformly, diminishing the stress concentrations. In highly orthotropic materials the use of tabs are essential. The specimen dimensions can be seen in Figure 1.

As said before, in tensile tests the tabs protect the specimen from the grips that applies a huge normal force so it doesn't slip. The high friction caused by this force is what makes it possible to apply the tension load on the specimen but it also creates interlaminar compressive stresses which can cause the result to be inaccurate.

The tabbed specimen were manufactured using 4 layers a 0.45 mm (0.017") thin woven carbon cloth in order to obtain 2 mm (0.08") final thickness (after surface grinding). Final manufactured specimen with tab can be seen in Figure 2.

The tabs must "match" the composite material used in the specimen, in terms of dimensions and the interaction between the adhesive and the two composites according to Adams (2002). For these tests, glass fiber tabs, with the same thickness, were used due to its high compression strength. Both the specimen and the tabs were laminated using epoxy resin, and then glued

Figure 1: Tabbed Specimen by ASTM D3039 Standard (Dimensions im mm)

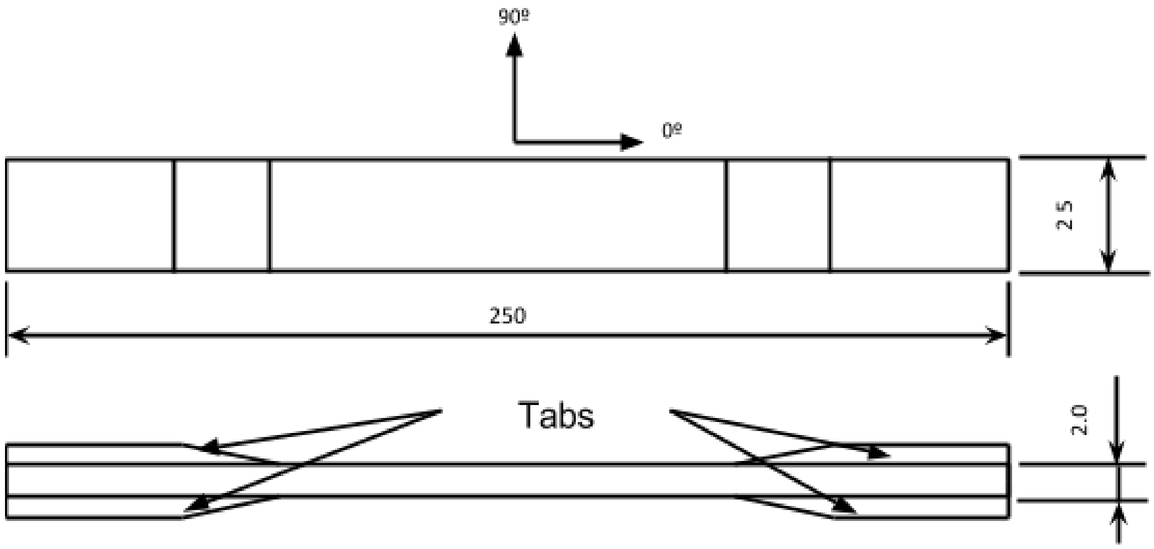


Figure 2: Glass Fiber Tabs Glued to the Specimen

together by the same resin. This variables might influence the result negatively, so a few remarks raised by Catalanotti *et al*. (2012) must be attended:

- The tab material must be less stiff than the specimen, but still be able to apply the grip load uniformly. Glass fiber has lower elastic modulus than the carbon fiber, so the laminate should be also less stiff.
- The tab chamfer diminishes discontinuities between the part section with the tabs and the section of the specimen, and so relieving this transient region of stress concentrations. However, the chamfer is not supported by the grip and when load is applied it flexes, causing tension loads in the surface that it's glued. The interlaminar tension strength is almost always low, because it depends on the matrix strength, and so, the tension loads might cause a delamination.
- The dimensions for tabs may differ depending on the material used. For this specimen made of carbon fiber and the tabs of glass fiber the thickness used was 1.5 mm and chamfer angle between 20º as recommended by some authors like Camanho *et al*. (2014).
- High strength adhesives (like epoxy) must be used in order to sustain great shear loads. The adhesive thickness should not be greater than 0.5 mm and can be controlled by measuring the total thickness of the specimen with tabs.

If any tab adherence or strength issue appears in the tensile test, the specimen thickness can be diminished in order to diminish the ultimate load to damage the laminate.

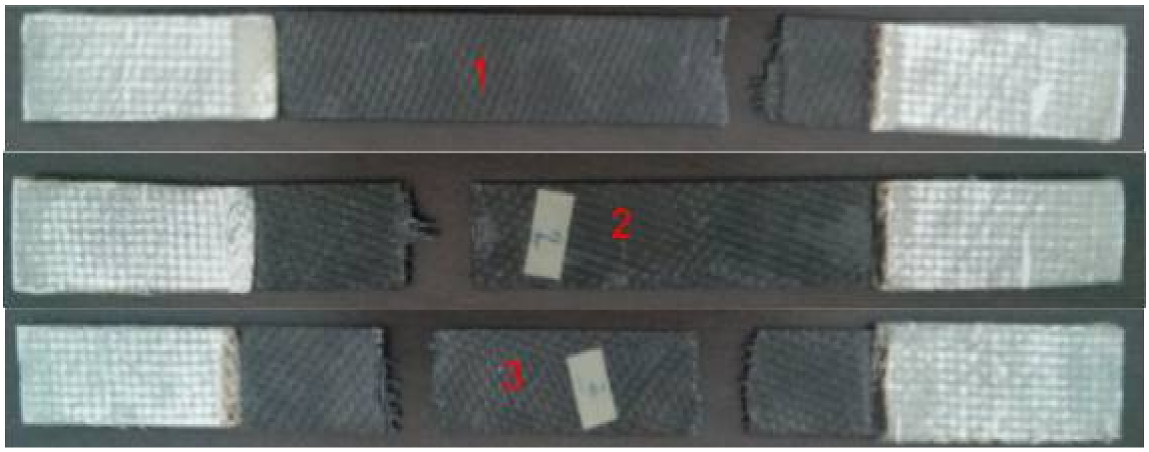
EXPERIMENTAL RESULTS

After applying traction loads to the specimen the following results were observed in Figure 3.

The angle of the fracture is 90° (transverse) to the axial loading. This happens because the woven used were biaxial, but 45° ply were not used. It's possible to see plies failing in different position (millimeters away from each other) which means that progressive failure analysis like Last Ply Failure (LPF) must be used for this study.

LPF method will make the software to consider the failure of the structure only when all the plies have a value of 1 for any of the failure modes. For example, this coupon made of 4 plies will only collapse (fail) when all plies fail in a given mode,

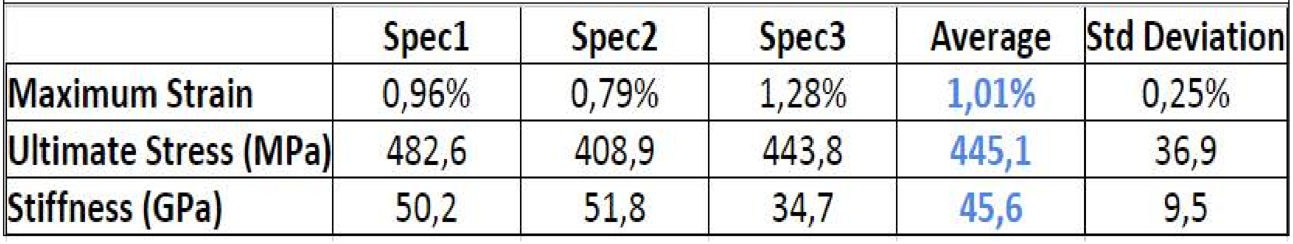
Figure 3: Test Specimens After Tensile Test



like fiber tension. That means some plies will have greater value than 1, meaning that they failed before the last one. Table 1 has the experimental results.

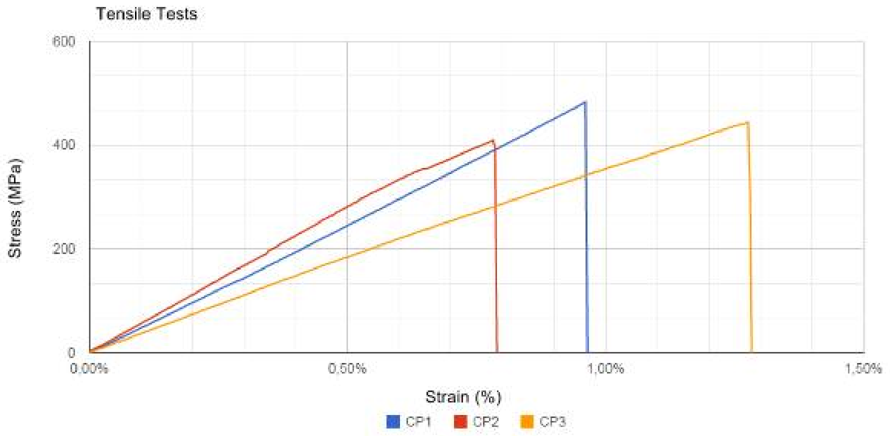
The results show that big differences may occur due to manufacturing variables. But the average values were used in to the FEA model and a minor error were detected, making the Hashin criteria a good correlation to real structures in plane stress. This will be discussed further. Figure 4 shows the tensile tests curves.

Table 1: Test Results



	Spec1	Spec2	Spec3	Average	Std Deviation
Maximum Strain	0,96%	0,79%	1,28%	1,01%	0,25%
Ultimate Stress (MPa)	482,6	408,9	443,8	445,1	36,9
Stiffness (GPa)	50,2	51,8	34,7	45,6	9,5

Figure 4: Tensile Tests Curves Stress x Strain



The tensile curves shows the big differences between specimens, but the ultimate stress (maximum error 8.3%) where close enough, and the most important aspect where the fragile behaviour of a carbon fiber/epoxy composite, with almost none plastic strain, which makes the hashin failure criteria ideal for this material application.

COMPUTATIONAL MODEL (FEA)

The coupon was modelled using shell elements since the Hashin criteria for ABAQUS considers only in plane stress elements, and this study's role is to demonstrate a good FEA to experimental correlation using less computational resources as possible.

The ply orientation is the same used to manufacture the specimens $(0°, 90°)_2$ which gives an orthotropic characteristic to the laminate. Figure 5 shows that.

To get reliable analysis, sometimes a fine mesh is needed, but as the scope of this study was to get good results with less computational resources a mesh convergence test was made before the actual analysis.

Figure 6 shows that less than 3% error wasdetected using the same variables and boundary conditions, but with different element size (the element type was the same), so the convergence was achieved. Therefore the final mesh uses 2x2 mm elements in the whole specimen and 0.1 mm elements where the model fails as shown in the Figure 7.

The load was applied by applying displacement to one edge of the part and constraining the other edge, as is in the real part. The machine applies traction loads by pulling the edges of the specimen. The displacement ratio input is the same of the experimental, 2 mm/min. FEA results are seen in Figures 8 and 9.

Ultimate Von Mises stress is used since the Hashin criteria states the behavior pre damage is elastic. Due to ply orientation, the stresses are uniform throughout the part as seen in test specimen #3, failing in two places. Result values obtained in this simulation are very close to the experimental tests. Strain at failure is 0.975% considering the displacement of 1.462 mm in 150 mm (free specimen length) which is also very close to experimental results.

Hashin's fiber tension failure criterion was satisfied (value equal to 1) since this kind of axial loading tends to load the fiber instead of the

Figure 5: Coupon Design and Ply Orientation

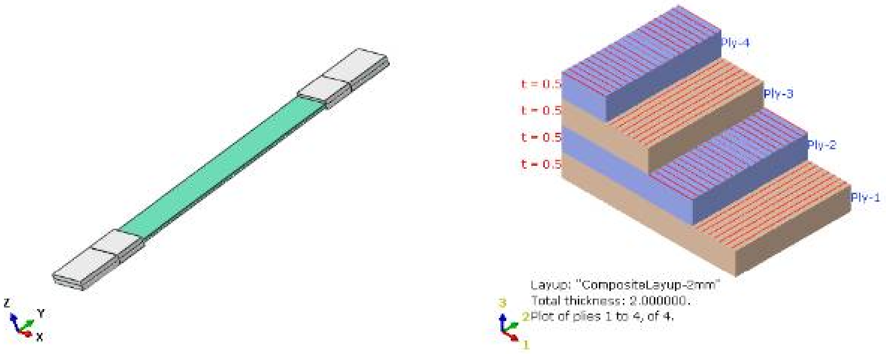


Figure 6: Mesh Convergence Test

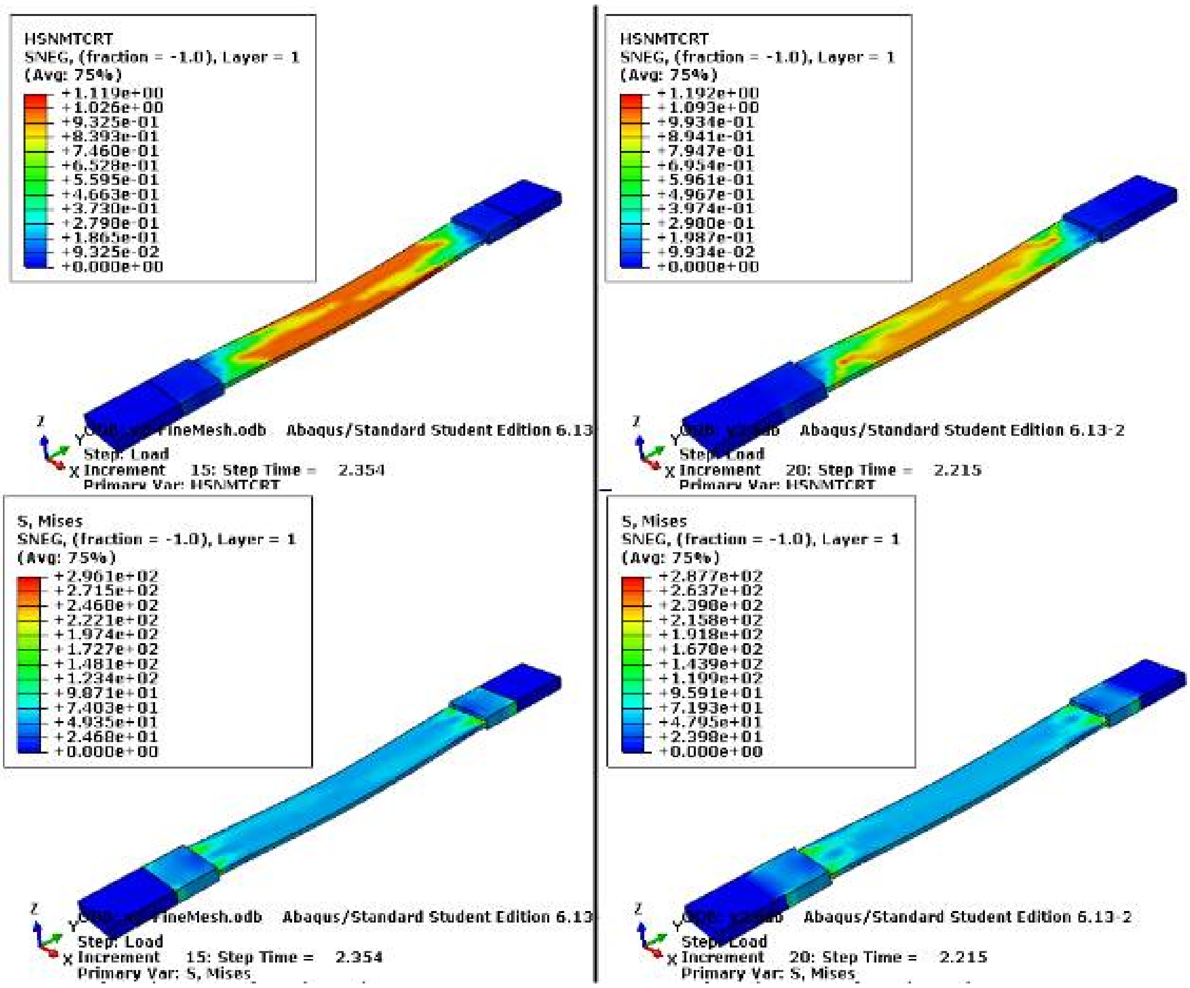


Figure 7: Meshed Specimen, Shell Elements (left) and Thickness Rendered (Right)

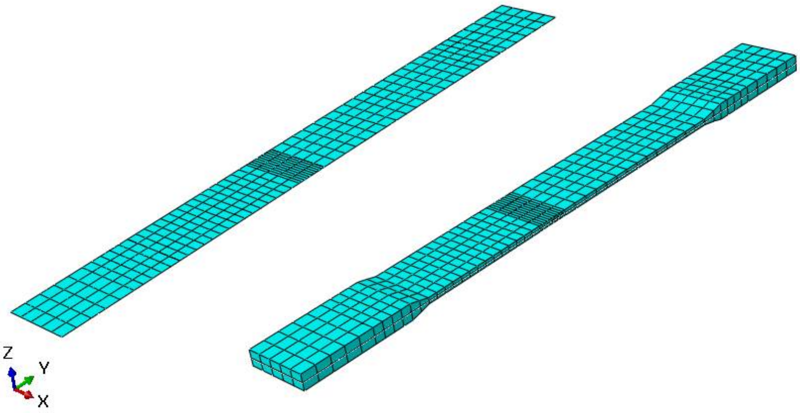


Figure 8: Ultimate Stress (439.8 MPa)

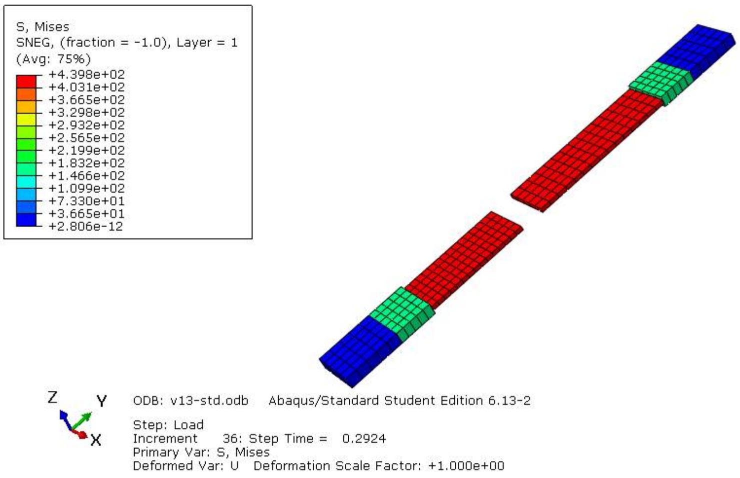
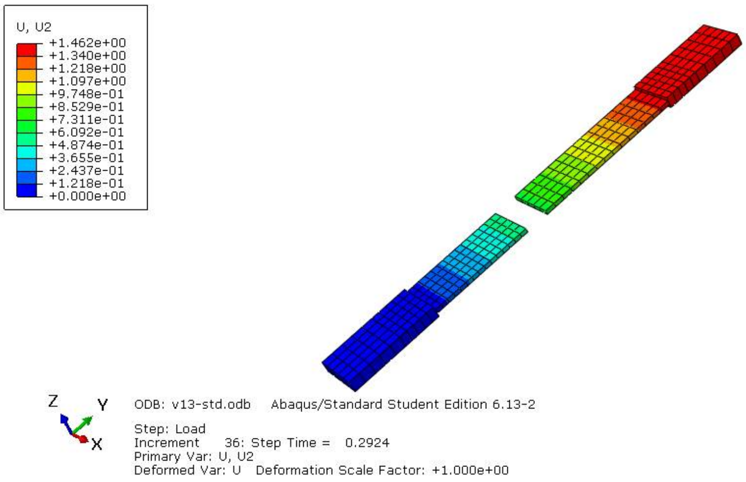


Figure 9: Displacement at Failure



matrix. Matrix tension criterion value was 1.28 x 10^{-5} mainly close to the ends of the tabs caused by the flexing of the tab chamfer as mentioned before. The other two criteria had very small values since no compression was simulated here. The compressive forces caused by the grips could be applied but wasn't in the scope of this study.

CONCLUSION

The numerical model could be correlated to the experimental tests which follows ASTM D3039 standard. Computational resources were minimized and the objective of this study satisfied. ABAQUS software has a variety of built-in tools, that can help create realistic models easily and predicts with a good accuracy stresses in composite structures. A very good correlation between numerical predictions and experimental results was obtained using the proposed failure model.

Further studies for tridimensional stresses are needed since the Hashin criteria solve equations for structures in plane stress. Considering many structures these days can be simplified to surface parts, these results are essential for time and cost reduction in the industry.

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