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Research Paper

## SYNCHRONIZING EQUIVALENT CLOCKS ACROSS INERTIAL FRAMES

Chandru Iyer<sup>1</sup> and G M Prabhu<sup>2\*</sup>

\*Corresponding Author: **G M Prabhu**, ✉ [prabhu@iastate.edu](mailto:prabhu@iastate.edu)

The second postulate of special relativity, namely, the equivalence of inertial frames, implies that all clocks must run identically across inertial frames. Under this principle, global clock synchronization may be feasible if an appropriate procedure can be developed. It is well-known that synchronization within an inertial frame using the methods of light rays or slow separation of clocks results in synchronization that is specific to that inertial frame. This paper describes a new procedure to synchronize clocks co-moving with different inertial frames, and analyzes its effectiveness.

**Keywords:** Special relativity, Lorentz transformation, Clock synchronization

### INTRODUCTION

Distances between spatial locations within an inertial frame are measured by an observer “by marking off his measuring-rod in a straight line as many times as is necessary to take him from the one marked point to the other. Then the number which tells us how often the rod has to be laid down is the required distance” (Einstein, 1961). It is also simple to measure the time interval between two events happening at the same location in an inertial frame by using a single clock present at that location. However, the measurement of a time interval between events taking place at different locations in an inertial

frame requires a multitude of clocks situated at various locations. The other option is to send a signal to a location where the reference or standard clock is present. The latter option is generally avoided because it involves knowing the distance between the locations as well as prior knowledge of the signal speed.

Synchronizing a number of clocks at one location and later separating them to different locations is another option. This option was acceptable under classical physics. However with the advent of special and general relativity this option has its limitations because of the effect of motion on clocks.

<sup>1</sup> Techink Industries, C-42, phase-II, Noida, 201305, India. E-mail: [chandru\\_i@yahoo.com](mailto:chandru_i@yahoo.com)

<sup>2</sup> Department of Computer Science, Iowa State University, Ames, IA 50011, USA.

Under the first postulate of special relativity, the signal speed of a light ray is constant in all inertial frames and therefore this has been used in thought experiments by many authors to synchronize clocks equidistant from a reference point by sending a light signal from that reference point (Bohm, 1965; Resnick, 1968).

The other possibility is to separate identical clocks very slowly with a limiting speed tending to zero, so that their running is not affected. This option was examined by Lorentz (Bohm, 1965, pp. 32-34) who demonstrated that even under slow separation, clocks in a 'moving' reference frame will become asynchronous, whereas they will remain synchronous in an inertial frame at 'rest'.

Both the above procedures give specific synchronicities that are unique to a given inertial frame but different for inertial frames in relative motion. Further, both procedures give identical results in a given inertial frame. The resultant synchronicity is widely known as standard Einstein synchronization (Ohanian, 2004). Reichenbach (1958, originally in 1927) had argued that this is only a conventional synchronicity and there is no compelling reason to adopt this particular synchronicity. He had proposed that alternate synchronicities can be developed by assuming different onward and return speeds for light without affecting causality. The Reichenbach synchronization, as it has been called in Ohanian (2004), has a parameter, epsilon,  $0 < \epsilon < 1$  and in this method, if the round trip speed of light is  $c$ , the onward speed is assumed to be  $c/(2\epsilon)$  and the return speed as  $c/[2(1 - \epsilon)]$ . The total round trip time ( $2s/c$ ) is thus divided into two parts  $[(2s/c)\epsilon]$  for the onward journey and  $[(2s/c)(1-\epsilon)]$  for the return journey. With the value of  $\epsilon = 1/2$ , the onward and return speeds of light

become identical and this leads to the Einsteinian synchronization. For other values of  $\epsilon$ , with the restriction that it is positive and less than 1, we get the Reichenbach synchronization (Ohanian, 2004).

Selleri (1996) has argued in favor of an absolute simultaneity. Rowland (2006), in his concluding remarks makes the observation that "a uniformly accelerating, effectively rigid rod only has instantaneous rest inertial frames, as one might expect it to, if inertial frames use Einstein synchronicity." However, he immediately adds that "while this observation provides yet another argument for accepting Einstein synchronicity as the 'natural' choice for a simultaneity convention, it is acknowledged that it does not in fact defeat the 'conventionality of simultaneity' thesis."

Ohanian (2004) has given a complete review of the debate on the conventions relating to synchronization. He also argues that the dynamical considerations forbid any synchronization other than the Einsteinian one, and if an inertial frame adopts a Reichenbach synchronization, Newton's laws would be violated. However, Martinez (2005) and Macdonald (2005) are not in complete agreement with Ohanian (2004).

Martinez (2005) has discussed the origin of the Einsteinian synchronization. He observes that the original German word '*festsetzung*' used by Einstein (1905) to prescribe the Einsteinian synchronization has been translated into English as 'stipulation' and into French as 'convention.' Eddington also advanced the concept that the Michelson-Morley experiment only determined the round trip speed of a light ray as a constant and a synchronization convention was needed to further specify that the speed of light remained constant

on both the onward and return trips (Martinez, 2005). Macdonald (2005) argues that Einstein definitely intended the synchronization proposed by him as a method or definition. And this is the reason Einstein emphasized that his definition “is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation (*festsetzung*) which I can make of my own free will in order to arrive at a definition of simultaneity.”

In his reply to the comments by Martinez (2005) and Macdonald (2005), Ohanian (2005) has argued that when the Einsteinian synchronization convention is adopted in all inertial reference frames, it “permits us to express the laws of physics in their simplest form.” He further states that “the adoption of a preferential inertial reference frame in which all the laws of physics take their simplest form compels the E (Einsteinian) synchronization and forbids the R (Reichenbach) synchronization” (Ohanian, 2005).

In this paper we propose a constructive procedure for synchronizing a three-clock system using the second postulate of special relativity. We assume that all clocks (even if in relative motion) run at the same rate. All the three clocks are under uniform relative motion in relation to each other and each one of them falls strictly under the purview of special relativity and the Lorentz transformations. The success of the procedure is checked using the Lorentz transformations and it is concluded that clocks in relative motion do not run identically.

## RELATION BETWEEN SIMULTANEITY AND LENGTH CONTRACTION

Time dilation is the phenomenon where the observed time rate of an observer’s reference frame is different from that of a different reference

frame. In special relativity, clocks that are moving with speed  $v$  with respect to an inertial system of observations are found to be running slower (Møller, 1952). The formula for determining time dilation in special relativity is:

$$\Delta t_0 = \Delta t \sqrt{1 - v^2 / c^2}, \text{ where}$$

$\Delta t_0$  is a time interval as measured with a ‘moving’ clock that is physically present at the two events under consideration,

$\Delta t$  is the same time interval as measured by another ‘stationary’ inertial frame with spatially separated clocks,

$v$  is the relative speed between the clock and the stationary system, and

$c$  is the speed of light.

The Lorentz transformations of spatial and temporal event coordinates between two inertial frames in relative motion ordain that a particular clock of one frame observed from another frame appears to run slow, and the set of clocks in one frame appears asynchronous as well as slowing down when viewed from the other frame. The asynchronicity and the slowing down seem to combine to create a symmetric perception of each other’s frame.

The question whether a moving clock runs slow or only appears to run slow is an intriguing one. For all practical purposes a moving clock runs slow. However, if an observer A is attached to the moving clock, his perception will be that the set of clocks in the inertial frame B that is observing him are asynchronous and for this reason B concludes that the moving clock A is slowing down. For the observer attached to the moving clock, the rate at which his clock is running is indeed the ‘correct’ rate, and any

conclusion to the contrary is due to improper synchronization, which indeed is the result of the slowing down of the clocks that are 'moving,' in B according to A's perception. According to Sears *et al.* (1980), there is no difficulty in synchronizing two clocks in the same frame of reference; only when a clock is moving relative to a given frame of reference do ambiguities of synchronization or simultaneity arise.

The perceived slowing down of clocks and possible asynchronicities between them also contribute to discrepancies in length measurements (Resnick, 1968). Consider a train moving at a velocity  $v$  and whose length is  $L$  as measured by observers on the train. A person on the platform measures the length of this train as  $L\sqrt{1-v^2/c^2}$ . Observers on the train explain this discrepancy by the 'errors' associated with the measurements made on the platform. They contend that: "A person on the platform stands at one location with a stop watch and measures the time elapsed between the passing of the two ends of the train at his location. Let this measurement be  $T$ . This person calculates the length of the train as  $vT$ . Since the clocks on the platform are running slow, he calculates a smaller value for length." However, observers on the platform have the following explanation to offer: "The length of the train was  $L$ , when it was stationary. While moving at  $v$ , it has contracted to  $L\sqrt{1-v^2/c^2}$ . Since all rulers on the train have also contracted by the same factor, the train continues to measure its length as  $L$ , which is in actuality  $L\sqrt{1-v^2/c^2}$  (while the train is moving)."

Thus we find that the observation of length contraction in moving frames is closely related to the observed slow running of clocks. It is also

worthwhile to note that a "moving" frame in spite of 'contracted' lengths and 'slow running' clocks measures the relative velocity correctly. The 'stationary' observers on the train explain this as follows. The apparent "movement" of a point object in the train's inertial frame by a distance  $x$  will be interpreted as a movement by a distance  $x' = x / \sqrt{1-v^2/c^2}$  by the platform due to the contraction of rulers in the platform. The time interval will be measured by spatially separated clocks on the platform as

$$T' = (x/v)\sqrt{1-v^2/c^2} + (vx)/(c^2\sqrt{1-v^2/c^2}).$$

The first term indicates the slow running of clocks on the platform and the second term indicates the asynchronicity in spatially separated clocks on the platform. Simplifying, we get

$$T' = (x/(v\sqrt{1-v^2/c^2}))$$

The platform correctly measures the relative speed  $v = x'/T'$ . According to observers on the train, observers on the platform wrongly measured both the distance and the time, but they correctly estimated the relative speed. Thus we find that the apparent asynchronicity and slow running of clocks in a moving frame is the cause of all discrepancies in length and time-interval measurements. But it also has the compensating effect of the relative velocity between the frames to be observed as the same value by both the frames.

## PROCEDURE TO SYNCHRONIZE A THREE CLOCK SYSTEM

We describe a three-clock system from some arbitrary inertial frame in the following fashion. Three identical clocks  $k$ ,  $m$  and  $n$  are in relative motion with velocities  $v$ ,  $u$ , and  $w$ , and at some instant appear as below:

$$k \rightarrow v \quad m \rightarrow u \quad n \rightarrow w$$

such that  $v > u > w$ . Furthermore we assume that the spatial separation of the clocks are such that the events  $E_1$  ( $k$  passing  $m$ ),  $E_2$  ( $k$  passing  $n$ ), and  $E_3$  ( $m$  passing  $n$ ) happen in the order  $E_1, E_2, E_3$ . We design our thought experiment so that when  $E_1$  occurs (that is, when  $k$  and  $m$  pass each other),  $m$  synchronizes its clock with  $k$ ; similarly when  $E_2$  occurs (that is, when  $k$  and  $n$  pass each other),  $n$  synchronizes its clock with  $k$ . Thus we presume that after the event  $E_2$ , both clocks  $m$  and  $n$  are also synchronized as they are both synchronized with clock  $k$ .

We would like to examine the correctness of this presumption by applying the Lorentz transformations and in particular by the actual observations of  $m$  and  $n$  as they pass each other at the occurrence of event  $E_3$ .

We denote the co-moving frames attached with the clocks  $k, m$ , and  $n$  as  $K, M$ , and  $N$ , respectively. For simplicity we take our inertial reference frame to be the co-moving frame  $N$  attached with clock  $n$ . Thus we have  $w = 0$ , and we assume the velocities of clocks  $k$  and  $m$  to be  $v$  and  $u$  respectively as observed by frame  $N$ .

### ANALYSIS OF THE PROPOSED SYNCHRONIZATION PROCEDURE

Let us assume that event  $E_1$  occurs at a distance  $s$  from clock  $n$  (in frame  $N$ ). At this event we synchronize clocks  $k$  and  $m$  so that  $t_k = 0$  and  $t_m = 0$ . Clock  $k$  will reach clock  $n$  (event  $E_2$ ) after a time of  $(s/v)$ .

However, clock  $k$  will show a time of  $t_k = (s/v)\sqrt{1-v^2/c^2}$  when it reaches clock  $n$  because of time dilation. According to the procedure set out in our thought experiment, we synchronize

clock  $n$  with clock  $k$  when they meet at event  $E_2$ .

Therefore, at event  $E_2, t_k = t_n = (s/v)\sqrt{1-v^2/c^2}$ .

According to frame  $N$ , at this time clock  $m$  would have traveled a distance  $u(s/v)$  and the distance remaining for clock  $m$  to reach clock  $n$  is  $(s - u(s/v))$ . This distance will be covered in a time interval of  $(s/u) - (s/v)$ . This time will be clocked by clock  $n$  between  $E_2$  and  $E_3$ , and thus at  $E_3$  clock  $n$  will read

$$t_n = [(s/u) - (s/v)] + [(s/v)\sqrt{1-v^2/c^2}].$$

When clock  $m$  reaches clock  $n$ , clock  $m$  will read  $t_m = (s/u)\sqrt{1-u^2/c^2}$ . This is because at  $E_1, t_m$  was 0 and the time taken by  $m$  between  $E_1$  and  $E_3$  is  $s/u$  (as observed by frame  $N$ ). This will be clocked as  $(s/u)\sqrt{1-u^2/c^2}$  by clock  $m$ . Thus the difference between clocks  $n$  and  $m$  when they meet at the occurrence of event  $E_3$  is

$$t_n - t_m = [(s/u) - (s/v) + [(s/v)\sqrt{1-v^2/c^2}] - [(s/u)\sqrt{1-u^2/c^2}].$$

The above quantity is not zero, indicating that  $t_n \neq t_m$ .

Since we specified the velocities of  $K$  and  $M$  with respect to  $N$  as  $v$  and  $u$  respectively, it was convenient to base our reference frame as  $N$  to arrive at the time difference between clocks  $n$  and  $m$ . If we base our considerations from any arbitrary frame instead of frame  $N$ , then by using the relativistic velocity addition formulae, it can be shown that the expression  $(t_n - t_m)$  remains the same in value; this is as it should be because this is the difference observed by clocks  $n$  and  $m$  at the same space-time point  $E_3$ , and any observation at the same space-time point is independent of the reference frame.

In the above analysis, apart from the relative velocities between the inertial frames, we have used 's', the distance (observed by frame  $N$ ) between clocks  $n$  and  $k$  at the occurrence of  $E_1$ , as a characterizing parameter of the system. We have given an alternative derivation in the appendix using the time shown by clock  $k$  at the occurrence of  $E_2$  as a characterizing parameter of the system. We note that the system has only one additional parameter (apart from the relative velocities between the inertial frames) and the analysis given in the appendix does not use any distance variable as a parameter. Furthermore, the analysis presented in the appendix does not use any one inertial frame as a preferred inertial frame. The results are shown to be identical by both the methods.

However, the merit of the analysis presented here is that it is simple, has minimal algebra, and is fully in accordance with the Lorentz transformations.

## DISCUSSION

In the thought experiment described in the Procedure to synchronize a Three Clock System, we have applied the principle of the equivalence of inertial frames and the exact algebraic formulations contained in the Lorentz transformations and reached an inconsistent situation. Since the Lorentz transformations are the only feasible formulation under actual or apparent equivalence of inertial frames, the thought experiment proves that inertial frames are not actually equivalent but only apparently equivalent.

The non-zero difference in time shown by clocks  $n$  and  $m$  when they meet can be explained

by assuming any one of the following statements:

1. Frame  $K$  is stationary and isotropic. Clocks  $m$  and  $n$  run slow with respect to  $K$ .
2. Frame  $M$  is stationary and isotropic. Clocks  $k$  and  $n$  run slow with respect to  $M$ .
3. Frame  $N$  is stationary and isotropic. Clocks  $k$  and  $m$  run slow with respect to  $N$ .
4. Any other arbitrary inertial reference frame  $S$  is stationary and isotropic. Clocks  $k$ ,  $m$ , and  $n$  run slow with respect to  $S$  as a function of their velocities.

We observe that in none of the above scenarios do clocks  $k$ ,  $m$ , and  $n$  run identically. So we may conclude that clocks in relative motion do not run identically. There are two possible consequences of this result. One possible consequence is that there exists a unique isotropic 'stationary' reference frame  $S$ , with respect to which physical processes and clocks run slow in all other inertial frames (which are in relative motion with respect to  $S$ ).

The other possible consequence is that clocks  $k$ ,  $m$  and  $n$  are traces on the space-time continuum. The three events  $E_1$ ,  $E_2$  and  $E_3$  are the intersection of these traces (like vertices of a triangle). This possibility visualizes any particular existence of a clock  $k$ ,  $m$  or  $n$  at a space-time point as a permanent etching on the space-time continuum. Here the temporal sequences are only an interpretation of a particular inertial frame and in the space-time continuum there is no specific sequence, either temporal or spatial.

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## APPENDIX

### Alternative Derivation with no Preferred Inertial Reference Frame

Let there be three inertial frames,  $K$ ,  $M$ , and  $N$  with origins  $O$ ,  $O'$ , and  $O''$  respectively. Let the event coordinates of any event be  $(x, t)$  in frame  $K$ ,  $(x', t')$  in frame  $M$ , and  $(x'', t'')$  in frame  $N$ . Let the event of the meeting of  $O$  and  $O'$  be  $E_1$ , that of  $O$  and  $O''$  be  $E_2$ , and that of  $O'$  and  $O''$  be  $E_3$ . Let the time order of occurrence of the three events be  $E_1, E_2,$  and  $E_3$ , in that order.

Let the velocity of frame  $K$  with respect to frame  $N$  be  $v$  and that of frame  $M$  with respect to frame  $N$  be  $u$ . We assume that  $v > u$ .

By the principle of the relativistic velocity addition formula, the velocity of frame  $K$  with respect to frame  $M$  is  $p = \frac{v-u}{1-\frac{vu}{c^2}}$  and the velocity of frame  $M$  with respect to frame  $K$  is  $-p$ .

Statement (AA): Let  $O$  and  $O'$  synchronize their clocks to  $t = t' = 0$  at event  $E_1$ .

Statement (BB): Let  $O$  and  $O''$  synchronize their clocks to  $t'' = t = t_0$ , where  $t_0$  is the time shown by a clock at  $O$  at the occurrence of event  $E_2$ . (Note that  $O$  does not alter its time.)

From statement (AA) we derive the transformation of event coordinates between frames  $K$  and  $M$  as shown in Equation (A1).

$$x' = \frac{x + pt}{\sqrt{1 - p^2/c^2}}; \quad t' = \frac{t + px/c^2}{\sqrt{1 - p^2/c^2}} \quad \dots(A1)$$

From statement (BB) we derive the transformation of event coordinates between frames  $K$  and  $N$  as shown in Equation (A2).

$$x'' = \frac{x + v(t - t_0)}{\sqrt{1 - v^2/c^2}}; \quad t'' - t_0 = \frac{(t - t_0) + vx/c^2}{\sqrt{1 - v^2/c^2}} \quad \dots(A2)$$

From Equation (A1),  $x$  and  $t$  can be written as shown in Equation (A3).

$$x = \frac{x' - pt'}{\sqrt{1 - p^2/c^2}}; \quad t = \frac{t' - px'/c^2}{\sqrt{1 - p^2/c^2}} \quad \dots(A3)$$

Substituting the values of  $x$  and  $t$  obtained from Equation (A3) into Equation (A2), the direct transformation between frames  $M$  and  $N$  are as shown in Equations (A4a) and (A4b).

APPENDIX (CONT.)

$$x'' = \frac{\frac{x' - pt'}{\sqrt{1 - p^2/c^2}} + v \left( \frac{t' - px'/c^2}{\sqrt{1 - p^2/c^2}} - t_0 \right)}{\sqrt{1 - v^2/c^2}} \quad \dots(A4a)$$

$$t'' = t_0 + \frac{\left( \frac{t' - px'/c^2}{\sqrt{1 - p^2/c^2}} - t_0 \right) + \frac{v}{c^2} \left( \frac{x' - pt'}{\sqrt{1 - p^2/c^2}} \right)}{\sqrt{1 - v^2/c^2}} \quad \dots(A4b)$$

The event E<sub>3</sub> is characterized by x' = 0 and x'' = 0. Substituting x' = 0 into Equation (A4b) we get

$$t'' = t_0 + \frac{\left( \frac{t'}{\sqrt{1 - p^2/c^2}} - t_0 \right) - \frac{v}{c^2} \left( \frac{pt'}{\sqrt{1 - p^2/c^2}} \right)}{\sqrt{1 - v^2/c^2}} \quad \dots(A5)$$

Let  $\gamma_v = \frac{1}{\sqrt{1 - v^2/c^2}}$  and  $\gamma_p = \frac{1}{\sqrt{1 - p^2/c^2}}$ . Substituting x' = 0 and x'' = 0 into Equation (A4a), we obtain after simplification,

$$t_0 = t'(1 - p/v)\gamma_p \quad \dots(A6)$$

Substituting the value of t<sub>0</sub> from Equation (A6) into Equation (A5), we get

$$t'' = t'(1 - p/v)\gamma_p + t' \left( \frac{\frac{1}{\sqrt{1 - p^2/c^2}} - (1 - p/v)\gamma_p - \frac{pv}{c^2\sqrt{1 - p^2/c^2}}}{\sqrt{1 - v^2/c^2}} \right)$$

$$= t'\gamma_p \left[ 1 - (p/v) + (p/v)\gamma_v - \frac{pv}{c^2}\gamma_v \right]$$

After simplifying, we obtain the ratio of t'' to t' as shown in Equation (A7).

$$\frac{t''}{t'} = \gamma_p \left( 1 - (p/v) + \frac{p}{v\gamma_v} \right) \quad \dots(A7)$$

APPENDIX (CONT.)

The right hand side of Equation (A7) is not equal to 1, indicating that  $t'' \neq t'$ . This result is independent of any chosen observing inertial frame. For example, if the analysis is carried out from frame  $N$ , the ratio of times is

$$\frac{t_n}{t_m} = (1 - u/v)\gamma_u + \frac{u}{v} \frac{\gamma_u}{\gamma_v} \tag{A8}$$

where  $\gamma_u = \frac{1}{\sqrt{1 - u^2/c^2}}$

Using the relativistic velocity addition formula,  $p = \frac{v-u}{1 - \frac{vu}{c^2}}$  and after simplification, it can be shown

that the expression on the right hand side of Equation (A8) is identical to the expression on the right hand side of Equation (A7). Hence the result in Equation (A7) is the same if the observations are made from frames  $K$ ,  $M$ ,  $N$ , or any other arbitrary inertial frame  $S$ .



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**Hyderabad, INDIA. Ph: +91-09441351700, 09059645577**

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