International Journal of Engineering Research and Science & Technology

ISSN : 2319-5991
Vol. 6, No. 2
May 2017

www.ijerst.com

Email: editorijerst@gmail.com or editor@ijerst.com

International Journal of Engineering Research and Science & Technology

ISSN 2319-5991 www.ijerst.com
Vol. 6, No. 2, May 2017

Research Paper

SIMULATIONS OF FLOWS THROUGH FRACTURE POROUS MEDIA USING LATTICE BOLTZMANN METHOD

Patel Kaushal[1]*

*Corresponding Author: Patel Kaushal ✉ patel_kaushal1@yahoo.com

There are many different forms of models available for the computational fluid dynamics that simulate the fracture model over the different computational speed. In the case of Lattice Boltzmann Model, is capable of recovering full Naiver - Stokes equation to good accuracy. The purpose of this study is to analyse the fluid flow inside the fractured media. Uncertainties involved in the understanding of fracture architecture and properties often propagate in the construction of fracture flow models. In this paper we simulate different fracture porous media. We shall consider a square sample of porous material and put across it a low Reynolds two-dimensional fluid flow driven by external pressure gradient. We use bitmap image for identify the boundary and the location of the impermeable material inside the disorder media. In solving this model we uses D2Q9 lattice property. The mathematical model is implemented in Matlab and constructed in such a manner that the various fracture medium would be easily simulated.

Keywords: Fractured porous media, Lattice boltzmann method, D2Q9 lattice

INTRODUCTION

This work is inspired from the wide complexity of the physical systems and consequently by the necessity to simplify their complexity into fundamental processes.

In recent years, the Lattice Boltzmann Method (LBM) has developed into an alternative and promising numerical scheme for simulating fluid flows and modelling physics in fluids. The scheme is particularly successful in fluid flow applications involving interfacial dynamics and complex boundaries. Unlike conventional numerical schemes based on discretization of macroscopic continuum equations, the lattice Boltzmann method is based on microscopic models and mesoscopic kinetic equations. The fundamental idea of the LBM is to construct simplified kinetic models that incorporate the essential physics of microscopic or mesoscopic processes so that the macroscopic averaged properties obey the desired macroscopic equations.

[1] Veer Narmad South Gujarat University, Surat, India.

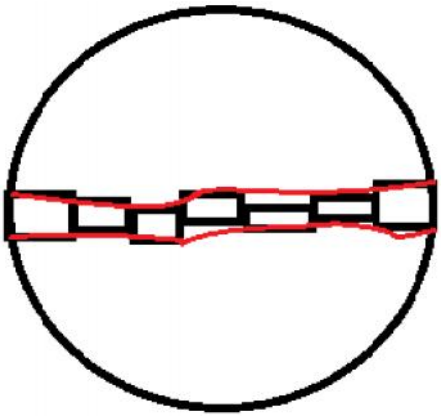
Historically originating from the Lattice Gas Automata (LGA) introduced by Frisch *et al.* (1986), the Lattice Boltzmann Equation (LBE) has recently become an alternative method for computational fluid dynamics. The essential ingredients in any lattice Boltzmann models which are required to be completely specified are: (i) a discrete lattice space on which fluid particles reside; (ii) a set of discrete velocities to represent particle advection; and (iii) a set of rules for the redistribution of particles residing on a node to simulate collision processes in a real fluid. Fluid-boundary interactions are usually approximated by simple reflections of the particles by solid interfaces.

In a hydrodynamic simulation using the lattice Boltzmann equation, one solves the evolution equations of the distribution functions of pretended fluid particles colliding and moving synchronously on a highly symmetric lattice space. The highly symmetric lattice space is a result of the discretization of particle velocity space and the condition for synchronous motions that is, the discretization of time and particle phase space are coherently coupled together. This makes the evolution of the lattice Boltzmann equation very simple, it consists in only two steps: collision and advection. One immediate limitation of the LBE method is due to its use of highly symmetric regular lattice mesh, which is usually triangular or square lattices in two dimensions and cubic in three dimensions. Obviously this is a serious obstacle to its applications in many areas of computational fluid dynamics. To deal with complex computational domains, various proposals have been made to use grids that are better suited to fit boundaries or to adapt meshes according to the physics of the system.

It has been shown recently that the lattice Boltzmann equation is indeed a special finite difference form of the continuous Boltzmann equation with some drastic approximations tailored for hydrodynamic simulations (He and Luo, 1997; *Phys. Rev. E*, 1997; *J. Stat. Phys*, 1997; and Abe, 1997). This makes the lattice Boltzmann method more amenable to incorporate body-fitted meshes (He and Doolen, 1997; and *J. Computational. Phys*, 1997) or grid refinement techniques (Filippova and Hanel, 1998). In most cases the regular lattice mesh is abandoned by decoupling the spatial-temporal discretization and the discrete velocity set, so that interpolations can be used in addition to the advection on a non-regular or non-uniform mesh.

In this work Lattice Boltzmann Equation is used to simulate the unsteady flow past through fractured porous media. Reservoir flow simulation has its roots in the microscopic description of fluid flow through particle collision. The mathematical fundamentals of reservoir simulators are based on the substitution of Boltzmann equation into the conservation of mass and momentum balance equation for a system and obtaining expressions for pressure in discrete grid-blocks within that system. The

Figure 1: Fracture Media and its Approximation



pressure equations contain bounce back condition that are related to the rock and fluid properties. One of the most important, and probably the most uncertain among all is called the transmissibility—a lumped parameter that retains information about the property of the porous medium, the fluid flowing through the medium, the direction of flow, and the position in space.

Because of the tortuosity of the flow paths, the actual fluid velocity will vary from point to point within the rock. However, in order to understand fluid flow in fractured media, we need to analyze its motion and hence have a realization of the variation of its velocity at a pore scale. Therefore, the equations used in our study must be different than the conventional reservoir simulation equations.

MATHEMATICAL MODELING

The motion of a continuous medium is governed by the principle of classical mechanics and thermodynamics for the conservation of mass, momentum, and energy. Fluids in a reservoir, as elsewhere, obey these principles; their flow can be modeled with equations that balance these inherent physical properties within a region of investigation. A brief discussion of the mathematical equations and the underlying assumptions that are generally used to describe the basic physics of fluid flow at a microscopic level in a medium is presented.

In this study, we will assume that the fluid is incompressible and Newtonian. Among the petroleum reservoir fluids, gas-free oil and water can be treated as incompressible fluids under typical reservoir conditions. The equation of motion for a viscous, incompressible, Newtonian fluid is given by the Navier-Stoke's equation:

$$\rho\left[\frac{\partial V}{\partial t} + (V \cdot \nabla)V\right] + \nabla P - \mu\nabla^2 V = \quad ...(1)$$

$$\nabla \cdot V = \quad ...(2)$$

where V and is the velocity vector and P is the pressure in the moving fluid at each point; is the external force per unit volume acting on the flowing fluid; ° and « are density and dynamic (or absolute) viscosity of the fluid, respectively; and ∇ and ∇^2 are the divergence and the Laplacian, operator respectively.

Equations (1) and (2) provides the motion of a (single-phase) fluid through porous-permeable media. Single-phase flow through fractures can also be modeled by this sets of equations. For some simple cases, such as the flow through a single fracture that can be approximated as flow through parallel plates, analytical solutions to the Stoke's equation exist. However, when the fracture geometry deviates from the idealization, as observed in the real reservoirs, numerical solutions to these partial differential equations are necessary. Numerical models then need to be constructed to simulate the flow and solve for the pressure and velocity variations with space and time within the fracture.

THEORETICAL BACKGROUND

Lattice Boltzmann Model, is capable of recovering full Naiver – Stokes equation to good accuracy so we use Lattice Boltzmann equation for solving above sets of equations.In that we use the kinetic theory as a fundamental of Lattice Boltzmann equation. In kinetic theory we consider the hard sphere model with elastic collision. The position and the momentum of the molecules are defined as $x = (x, y, z)$ and $p = (p_x, p_y, p_z)$. Then $f(x, p, t)$ is the probability density function for the presence of molecule in nine – dimensional phase space.

The probable no of molecules with position coordinates in the range $x \mapsto x \pm dx$ and momentum coordinates $p \longmapsto p \pm dp$ is given by $f(x, p, t)dxdp$. We introduce an external force F that is small relative intermolecular forces. If there are no collisions, then at time $t + dt$, the new positions of molecules starting at x are $x + \left(\frac{dx}{dt}\right)dt = x + dx$ and the new momentum is $p + Fdt = p + \left(\frac{dp}{dt}\right)dt = p + dp$. Thus, when the position and momentum are known at a particular time t, incrementing them allows us to determine at a future time. Then the updated particle velocity distribution function is defined as $f(x + dx, p + dp, t + dt)$. If the collision in not occurs then $f(x + dx, p + dp, t + dt) - f(x, p, t) = 0$ the particle velocity is not change then and if the collision occur then there is a change in velocity distribution function and it is described as $f(x + dx, p + dp, t + dt) - f(x, p, t) \neq 0$. The Boltzmann Equation Expresses a balance between transport of a particle and collision between particles then

$$\frac{Df}{Dt}_{Transport} = \frac{Df}{Dt}_{Collision}$$

$$\therefore \frac{\partial f}{\partial t} + p_\alpha \frac{\partial f}{\partial x_\alpha} = C(f) \quad ...(3)$$

where C(*f*) models the pair wise collision between particles.

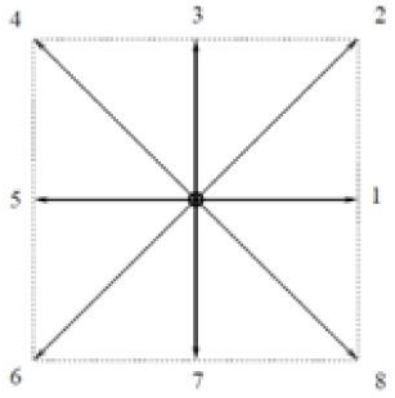
C(*f*) is evaluated by the Tailor's expansion of the updated velocity distribution function

$$x\frac{\partial f}{\partial x} + dp\frac{\partial f}{\partial p} + dt\frac{\partial f}{\partial t} + \frac{(dx)^2}{2}\frac{\partial^2 f}{\partial x^2} + \frac{(dp)^2}{2}\frac{\partial^2 f}{\partial p^2} + \frac{(dt)^2}{2}\frac{\partial^2 f}{\partial t^2} + \frac{dxdp}{2}\frac{\partial^2 f}{dxdp}\frac{dpdt}{2}\frac{\partial^2 f}{dpdt} + \frac{dtdx}{2}\frac{\partial^2 f}{dtdx} = \Gamma^+ - \Gamma^- \quad ...(4)$$

$$dx\frac{\partial f}{\partial x} + dp\frac{\partial f}{\partial p} + dt\frac{\partial f}{\partial t} + \frac{(dx)^2}{2}\frac{\partial^2 f}{\partial x^2} + \frac{(dp)^2}{2}\frac{\partial^2 f}{\partial p^2} + \frac{(dt)^2}{2}\frac{\partial^2 f}{\partial t^2} + \frac{dxdp}{2}\frac{\partial^2 f}{dxdp}\frac{dpdt}{2}\frac{\partial^2 f}{dpdt} + \frac{dtdx}{2}\frac{\partial^2 f}{dtdx} = -\frac{f - f^{eq}}{\tau} \quad ...(5)$$

Then the equilibrium function is defined as

Figure 2: The Lattice Pattern of the D2Q9 Model



$$f^{eq} = \rho w_i \left[1 + \frac{3}{c^2}c_i u + \frac{9}{2c^4}(c_i u)^2 - \frac{3}{2c^2}u^2\right] \quad ...(6)$$

where w_i, the weight factor corresponds to set of particles for D2Q9 lattice for rest particle $w_0 = \frac{4}{9}$, nearest neighbors have $w_i = \frac{1}{9}$ for $i = 1,3,5,7$ and farthest neighbors having $w_i = \frac{1}{36}$ for $i = 2,4,6,8$. c_i is the discrete sets of velocities for D2Q9 lattice and is defined as $c_0 = 0$, nearest neighbors $c_i = c\left(\cos(i-1)\frac{\pi}{4}, \sin(i-1)\frac{\pi}{4}\right)$ for and farthest neighbors $c_i = \sqrt{2}c\left(\cos(i-1)\frac{\pi}{4}, \sin(i-1)\frac{\pi}{4}\right)$ for $i = 2,4,6,8$. We take the speed of light in the media $c = \sqrt{3w_0}\frac{\Delta x}{\Delta t}$ where $\Delta x \; and \; \Delta t$ is the lattice spacing and time spacing respectively. For these models we take single time relaxation parameter $\tau = 1$.

The macroscopic variables are defined as a velocity moments of the velocity distribution function and that can be utilize the calculate velocity Equation (8) and density Equation (9) over the domain.

Mass density: $\rho(x,t) = m\int f(x,p,t)d^3p$ ...(7)

Flow velocity: $u(x,t) = \frac{m}{\rho}\int p\, f(x,p,t)d^3p$ ...(8)

Temperature: $T = \frac{m}{2R\rho} \int |p|^2 f(x,p,t) d^3p$ ...(9)

Stress Tensor: $p_{\alpha,\beta} = m \int f(x,p,t)(p_\alpha - u_\beta) f(x,p,t) d^3p$

RESULTS AND DISCUSSION

Table describe the numerical results of the flow past through disorder porous media. It is clear the LBGK model have been successfully simulate the flow in disorder media and it revealing the models capability and limitations. The disorder media model is capable of handling fluid flow around many objects in physically accurate manner. The velocity profiles of the fluid flow around the objects turnout to be very accurate. The bounce back condition allows for manifestation of boundary layer along the surface of the objects, which is a physically accurate description of fluid flow.

Table: Simulation of Various Fracture Models with Flow Velocity Vector at Grid Points

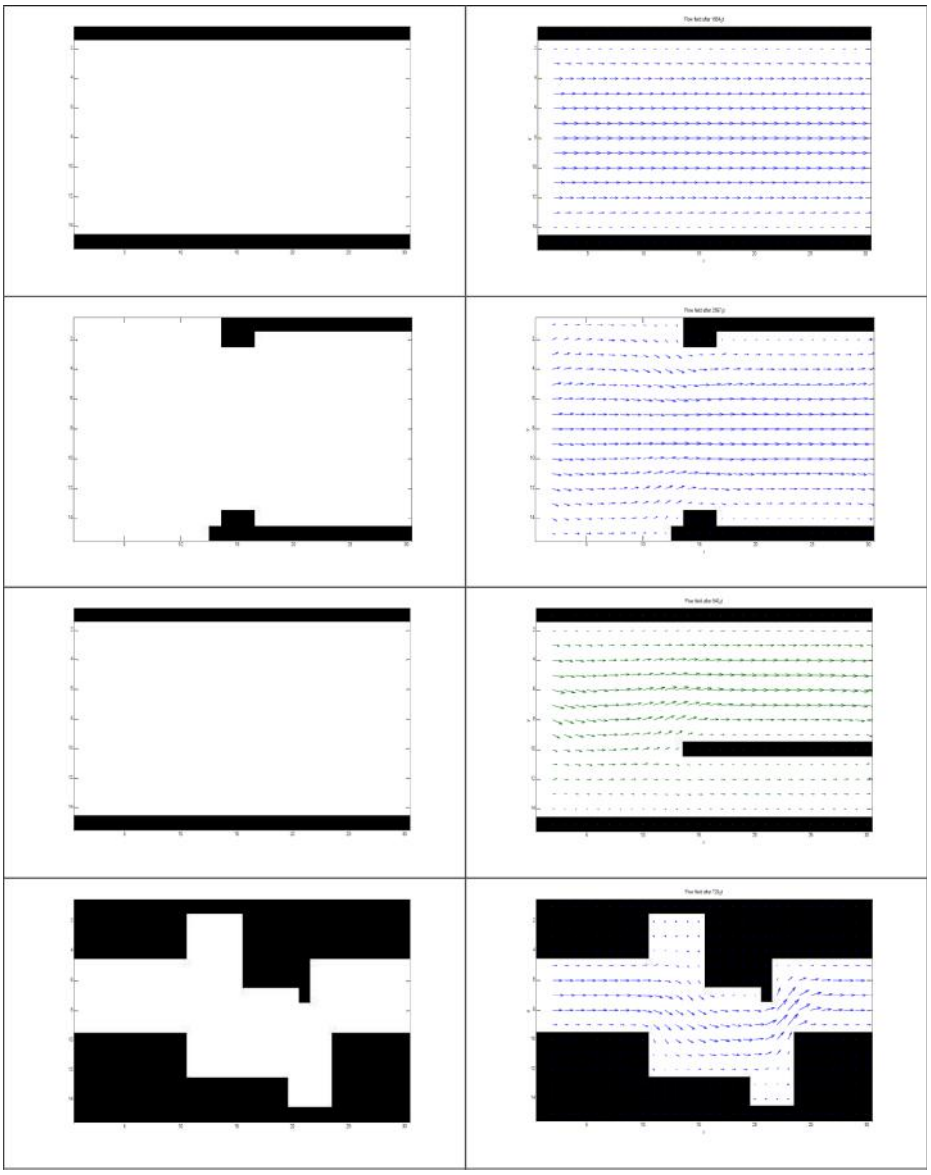
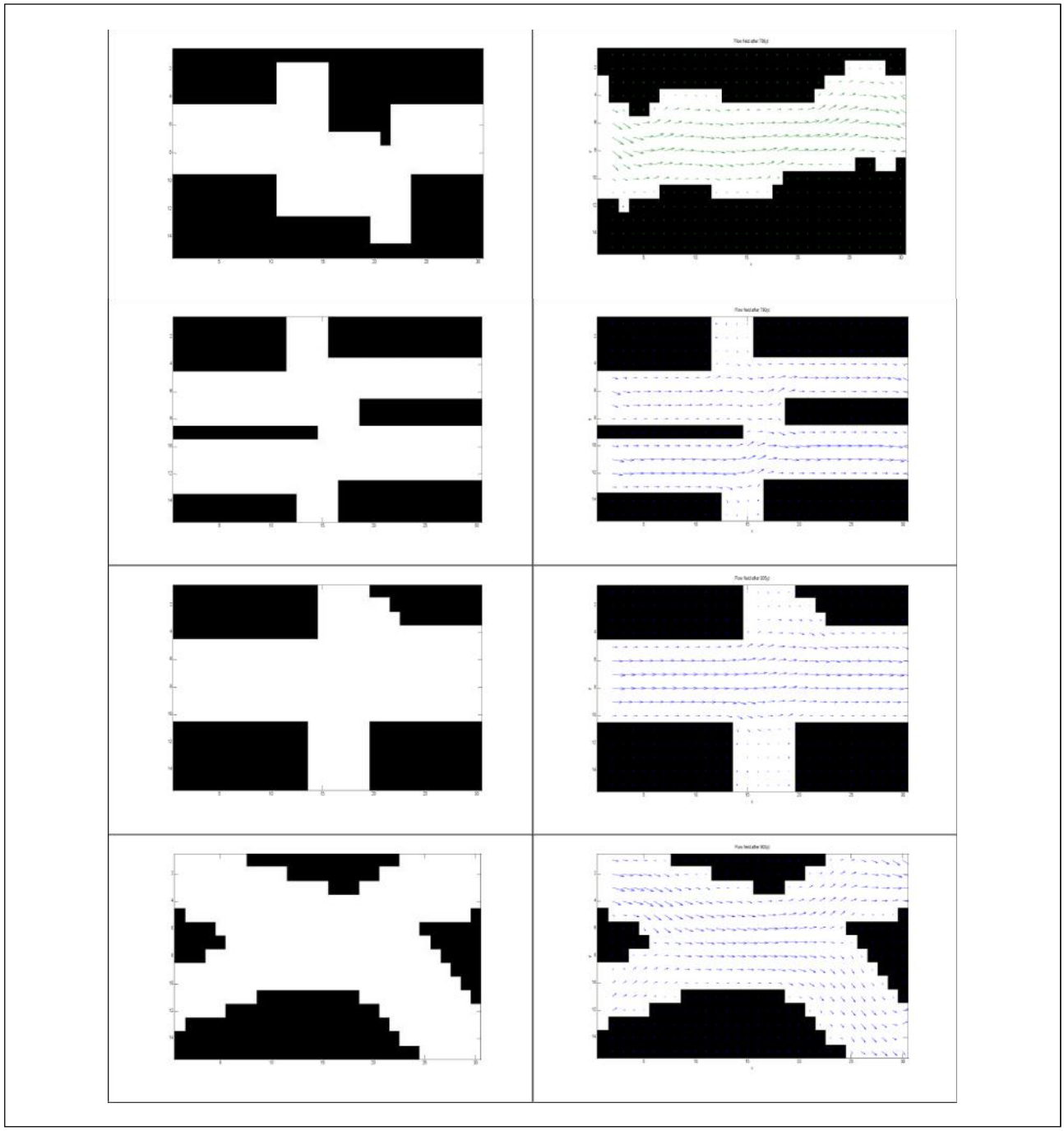


Table (Cont.)



CONCLUSION

A lattice Boltzmann equation was successfully applied to simulate flow through fractured porous media. The result obtain by the model is highly accurate in turs of physical behavior of the flow in fracture porous media. As compared to the traditional method available LBM is quite useful for predicting the flow characteristic in real phenomena. For the comparison with the traditional method, LBM can provide the higher order of accuracy for unsteady flow but require reasonable computational effort. The major

advantage of model is to it can be applicable to vide rage of complex geometry.

We have simulated two-dimensional single-phase fluid flow for eight different fractures. The computer program Matlab meets both computational and visualization needs of important fluid flow properties at micro-scale. The simple examples provide good insight into fluid flow in fractures and its dependence of fracture properties.

REFERENCES

1. "Lattice Boltzmann Method on Curvilinear Coordinates System: Flow Around a Circular Cylinder", *J. Computational. Phys*., Vol. 134, pp. 306-315.
2. "Lattice Boltzmann Model for the Incompressible Navier-Stokes Equation", *J. Stat. Phys*., Vol. 88, pp. 927-944.
3. "Theory of the Lattice Boltzmann Method: From the Boltzmann Equation to the Lattice Boltzmann Equation", *Phys. Rev. E*, Vol. 56, pp. 6811-6817.
4. Abe T (1997), "Derivation of the Lattice Boltzmann Method by Means of the Discrete Ordinate Method for the Boltzmann Equation", *J. Computational. Phys*., Vol. 131, pp. 241-246.
5. Bouzidi M, Dominique D and Lallemand P (2002), "Lattice Boltzmann Equation on a 2D Rectangular Grid", NASA/CR-2002-211658, ICASE Report No. 2002-18, June.
6. Bouzidi M, Firdaouss M and Lallemand P (2001), "Momentum Transfer of a Lattice-Boltzmann Fluid with Boundaries", *Phys. Fluids*, Vol. 13, pp. 3452-3459.
7. Chen H, Ctten S and N'iatthaeus W H (1992), "Recovery of the Navier-Stokes Equations Using a Lattice Gas Boltzmann Method", *Phys. Rev. A*, Vol. 45, pp. R5339-R5342.
8. Filippova O and Hanel D (1998), "Grid Refinement for Lattice-BGK Models", *J. Computational. Phys*., Vol. 147, pp. 219-228.
9. Frisch U, Hasslacher B and Pomeau Y (1986), "Lattice-Gas Automata for the Navier-Stokes Equation", *Phys. Rev. Lett*., Vol. 56, pp. 1505-1508.
10. He X and Doolen G D (1997), "Lattice Boltzmann Method on a Curvilinear Coordinate System: Vortex Shedding Behind a Circular Cylinder", *Phys. Rev. E*, Vol. 56, pp. 434-440.
11. He X and Luo L (1997), "A Priori Derivation of the Lattice Boltzmann Equation", *Phys. Rev. E*, Vol. 55, pp. R6333-R6336.
12. Karlin I V, Succi S and Orszag S (1999), "Lattice Boltzmann Method for Irregular Grids", *Phys. Rev. Lett*., Vol. 82, pp. 5245-5248.
13. Lallemand P and Luo L S (2000), "Theory of the Lattice Boltzmann Method: Dispersion, Dissipation, Isotropy, Galilean Invariance, and Stability", *Phys. Rev. E*, Vol. 61, pp. 6546-6562.
14. Latt J (2013), "The Lattice Boltzmann Method: Complex CFD and Massive Parallelism", March 4, NASA Glenn Research Center Cleveland, Ohio.
15. Qian Y H, D'humires D and Lallemand P (1992), "Lattice BGK Models/or Navier-Stokes Equation", *Europhys. Letter*, Vol. 17, pp. 479-484.
16. Schxfer M and Turek S (1996), "Benchmark

Computations of Laminar Flow Around a Cylinder, Notes in Numerical Fluid Mechanics", Vol. 52, pp. 547-566.

17. Sukop M and Thorne D (2006), "Lattice Boltzmann Modeling—An Introduction for Geoscientist and Engineers", Springer, New York.

International Journal of Engineering Research and Science & Technology
Hyderabad, INDIA. Ph: +91-09441351700, 09059645577
E-mail: editorijlerst@gmail.com or editor@ijerst.com
Website: www.ijerst.com