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Research Paper

RELIABILITY BASED OPTIMUM DESIGN OF A SIX SPEED GEAR BOX

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Reliability is the probability that a system, component or device will perform without failure for a specified period of time under specified operating conditions. The concept of reliability is of great importance in the design of various machine members. Conventional engineering design uses a deterministic approach. It disregards the fact that the material properties, the dimensions of the components and the externally applied loads are statistical in nature. In conventional design these uncertainties are covered with a factor of safety, which is not always successful. The growing trend towards reducing uncertainty and increasing reliability is to use the probabilistic approach. In the present work a six speed gear box are designed using reliability principles. For the specified reliability of the system (Gear box), component reliability (Gear pair) is calculated by considering the system as a series system. Design is considered to be safe and adequate if the probability of failure of gearbox is less than or equal to a specified quantity in each of the two failure modes. A computer program is developed in JAVA language to calculate the face widths in bending and surface failure modes. The larger one out of the two values is considered. By changing the variations in the design parameters, variations in the face widths are studied.

Keywords: Reliability based optimum Design, Six Speed Gear Box, Reliability principles

INTRODUCTION

Due to the increasing complexity of modern engineering systems, the concept of reliability has become a very important factor in the overall system design. As all the engineering systems are expected to perform satisfactorily over the duration of their expected life span the field of reliability based design has come into greater prominence in recent years. No system will perform reliably unless it is designed specifically for reliability. The reliability can be viewed as a measure of successful performance of the system.

The discipline of reliability engineering basically is a study of causes, distribution and prediction of failures. With increasing concerned over minimizing the cost of failure, reliability based design approach has become more important. In case of large systems reliability plays a crucial role. These systems are composed of several

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sub systems and components. All the individual components must be designed properly to achieve the specified overall reliability criteria.

A mechanical component is considered to have failed when it ceases to function properly for its intended use. In the present work critical machine element like gears are designed using reliability based design and the design parameters are calculated for the specified reliability. Design and development costs will increase with improved reliability because of the need to be more critical in the design and the need for more exhaustive testing of the equipment production cost also increase with the improved reliability because of the use of better components and closer control of process, inspection and test procedures. On the other hand costs of maintenance and repair fall although in many cases the only part of these that devolve on the manufactures are those that occur during the guarantee period, nevertheless good reliability represents an intangible asset in the way of continued or increased sales.

PROBABILISTIC DESIGN

Probabilistic design is a concept where by engineering variables are treated statistically. Each variable is modeled to reflect a spectrum of possible values. Classical equations are then adopted to yield meaningful results in terms of probability of failure.

In probabilistic design, the power transmitted, speed of the input gear, allowable and induced stresses in bending and surface wear and center distance of the gear pair are assumed as random variables. All the random variables are assumed to follow distribution. Deterministic data like number of teeth on gears (T_w, T_p), K_e, K_d, E, module and pressure angle are assumed to be known.

In the present work probability of failure of a gear is defined as the probability that induced bending stress or wear stress exceeds the corresponding strength of the material. Two failure modes namely the bending and surface wear modes are considered. The design parameter is the face width of gear is taken as larger one out of the two values obtained in bending and surface wear failure modes.

Approximate mean, standard deviation and coefficient of variation of induced shear stress can be obtained from relationships

$$f \approx f\left(\bar{x}_{1,2,\ldots n}\right)$$

and $\sigma_f \approx \left[\sum_{x=1}^{n}\left(\partial f/\partial x_i\right)^2\left(\sigma x_i\right)^2\right]^{1/2}$

Which holds when the dispersion of each random variable, $C = \sigma_x\sqrt{x}$ is less than 0.2.

FAILURE DUE TO BENDING STRESS

The mean and standard deviation of s_b are given by

$$\bar{S}_b = \beta\left(\bar{M}_t\right)/\left(\bar{A}\bar{t}_1\right) \quad \text{...(1)}$$

$$\sigma_{sb} = \left(1/\overline{At}\right)\left[\beta^2\bar{M}_t^2\left(\bar{t}^2\sigma_A^2+\bar{A}^2\sigma_t^2\right)+\beta^2\bar{A}^2\bar{t}^2\sigma_{Mt}^2/\bar{A}^2\bar{t}^2\right]^{1/2}$$

where

$$\sigma_{Mt} = \frac{97500}{\bar{n}_w}\sqrt{\left[\bar{P}^2\sigma_{nw}^2+\bar{n}_w^2\sigma_p^2\right)/\bar{n}_w^2\Big]}$$

The coefficient of variation of torque and induced bending stress can be expressed as:

$$C_{Mt}^2 = C_p^2 + C_{mw}^2$$

$$C_{sb}^2 = C_t^2 + C_A^2 + C_{Mt}^2$$

FAILURE DUE TO WEAR STRESS

The mean and standard deviation of s_w are given as

$$\bar{s}_w = \gamma\sqrt{(\bar{M}_t / \bar{t}) / A} \quad ...(2)$$

$$\sigma_W = \frac{(1/A\bar{t})\left[\left[4\bar{M}_t^2\gamma^2\bar{t}^2\sigma_A^2 + \gamma^2\bar{A}^1 M_t^2\sigma_t^2 + \bar{A}^2\bar{t}^2\gamma^2\sigma\bar{M}_t^2\right]^{1/2}\right]}{\left(4\bar{M}_t\bar{A}^2\bar{t}^2\right)^{1/2}}$$

The coefficient of variation of induced wear stress can be expressed as

$$C_{sw}^2 = \frac{C_t^2 + C_{Mt}^2 + 4C_A^2}{4}$$

Standard normal variate in bending mode and surface wear mode are

$$Z_b = -\left[\bar{S}_b - \bar{S}_b\right] / \left[\sigma_{sb}^2 + \sigma_{sb}^2\right]^{1/2}$$

$$Z_w = -\left[\bar{S}_w - \bar{S}_w\right] / \left[\sigma_{sw}^2 + \sigma_{sw}^2\right]^{1/2}$$

By rearranging the standard normal variate equations introducing the coefficient of variations, the mean values of induced bending and surface wear stress are

$$\bar{s}_b = \frac{\bar{S}_b \pm \left[\bar{S}_b^2 - \bar{S}_b^2(1 - Z_b^2 C_{sb}^2)(1 - Z_b^2 C_{sb}^2)\right]^{1/2}}{(1 - Z_b^2 C_{sb}^2)}$$

$$\bar{s}_w = \frac{\bar{S}_w \pm \left[\bar{S}_w^2 - \bar{S}_w^2(1 - Z_w^2 C_{sw}^2)(1 - Z_w^2 C_{s2}^2)\right]^{1/2}}{(1 - Z_w^2 C_{wb}^2)}$$

By taking the smaller values of s_b, Z, s_w the mean values of t1 and t2 can be calculated by using equation (1) and equation (2).

For the specified reliability of system (Gear box), component reliability (Gear pair) is calculated by considering the system as a series system.The face width of each gear pair is calculated for different output speeds and maximum out of these from two failure modes is considered as the face width of the gear pair.A program is developed in 'JAVA' to calculate the face width of the gear pair using the above two equations.

SIX SPEED GEAR BOX

While designing the following data is considered:

Power=10 HP, N1=100 rpm, S_w=17500 kg/cm^2, S_b=2500 kg/cm^2 and other data is assumed same as that of 4 speed gear box.

While designing a six speed gear box the following data is considered:

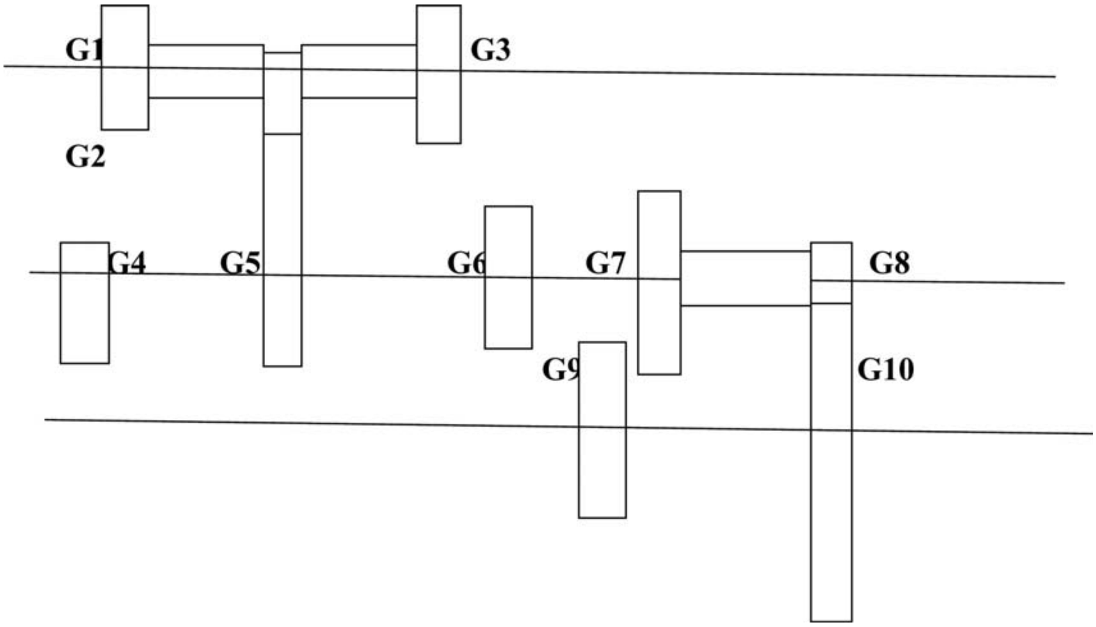
Power=10 HP,S_w=17500 kg/cm^2, S_b=2500 kg/cm^2 and other data is assumed same as that of 4 speed gear box. Speeds are 100 rpm, 140 rpm, 200 rpm, 280 rpm, and 400 rpm.

Number of teeth of gear pairs:

Gear pair	t_w	t_p
(G1,G4)	35	25
(G2,G5)	40	20
(G3,G6)	30	30
(G7,G9)	38	38
(G8,G10)	56	20

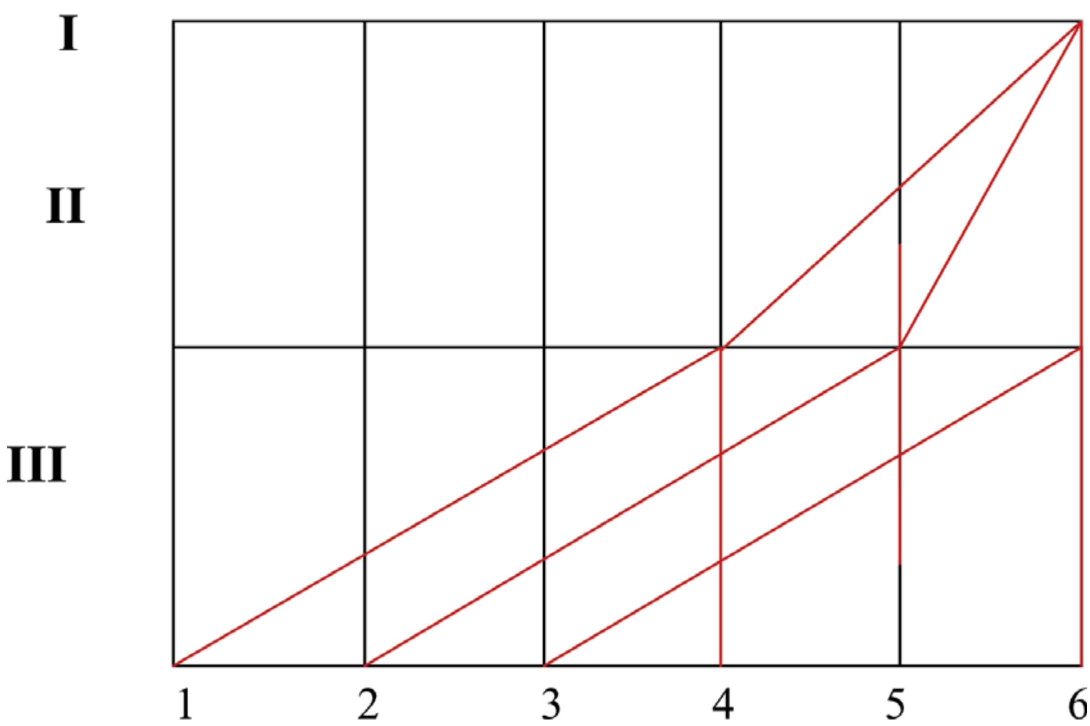
Kinematic Arrangement of Gear Box

Figure 1: Kinematic Arrangement of 6 Speed Gear Box



Ray Diagram

Figure 2: Ray Diagram of 6 Speed Gear Box



CALCULATION OF SPEEDS

N1=100 rpm

N2=N1x1.4=140 rpm

N3=N2x1.4=200 rpm

N4=N3x1.4=280 rpm

N5=N4x1.4=400 rpm

N6=N5x1.4=560 rpm

CALCULATION OF NUMBER OF TEETH

Stage: I

$T_a+T_b= T_c+T_d= T_e+T_f$

For gear pair (G2, G5):

$$\frac{N_b}{N_a}=\frac{T_a}{T_b}$$

$$\frac{T_a}{T_b}=\frac{280}{560}=\frac{1}{2}$$

Assuming minimum number of teeth as 20 i.e., T_a=20

T_b=40

For gear pair (G1, G4):

$$\frac{N_d}{N_c}=\frac{T_c}{T_d}$$

$$\frac{T_c}{T_d}=\frac{400}{560}=\frac{1}{1.4}$$

$$T_c+T_d=(T_a+T_b)=60$$

Therefore Tc=25 and T_d=35

For gear pair (G3, G6):

$$\frac{N_f}{N_e}=\frac{T_e}{T_f}$$

$$\frac{T_e}{T_f}=\frac{560}{560}=1$$

and $T_e+T_f= T_a+T_b$

$T_e=T_f=30$

Stage:II

Considering the transmission between the intermediate and output shafts.

$$T_g+T_h=T_i+T_j$$

For gear pair (G8,G10):

$$\frac{N_h}{N_g}=\frac{T_g}{T_h}$$

$$\frac{T_g}{T_h}=\frac{100}{280}=\frac{1}{2.8}$$

Assuming T_g=20

$$\frac{20}{T_h}=\frac{1}{2.8}$$

T_h=56

For gear pair (G7, G9):

$$\frac{N_j}{N_i}=\frac{T_i}{T_j}$$

$$\frac{T_i}{T_j}=\frac{280}{280}=1$$

$T_i = T_j = 56$

CALCULATION OF TORQUE

$$\text{Mt1}=\frac{HP\times 4500}{2\times\pi\times N1}=\frac{10\times 4500}{2\times\pi\times 100}=71.62\,kg-m$$

Mt2=$\frac{10\times4500}{2\times\pi\times140}=51.2kg-m$

Mt3=$\frac{10\times4500}{2\times\pi\times200}=35.8kg-m$

Mt4=$\frac{10\times4500}{2\times\pi\times280}=25.6kg-m$

Mt5=$\frac{10\times4500}{2\times\pi\times400}=17.9kg-m$

Mt6=$\frac{10\times4500}{2\times\pi\times560}=12.8kg-m$

CALCULATION OF MODULE

Maximum torque=71.62 kg-m

Maximum torque ray provides the value of the force acting at the gear teeth.

$$F=\frac{Mt_{max}}{m\times\frac{T}{2}}$$

$$F=\frac{143240}{56m}$$

$$F=\frac{143240}{56m}\text{ x } Y.C$$

Assuming Y=10, C=2 for mild steel gears

$$F=\frac{143240}{56m}=m^2\times20$$

$$m^3=\frac{143240}{56\times20}$$

m=5.038 mm

Standard module m=5 mm

DETERMINISTIC DESIGN

Failure Due to Bending Stress

The allowable stress in bending is

$s_b = S_b$ /F.S

Induced bending stress assuming the tooth as a cantilever beam with end load is

$S_b = \beta M_t / At_1$ ……….(1)

Where

M_t =72735 p/N_w

β = [$k_c k_d$ (i+1)] / i my cos α

k_c=1.5, α=20^o and k_d=1.1

y = 0.52(1+ 20/t_w)

A = (t_w + t_p) m / 2

The face width (t_1) due to bending is calculated as

$t_{1\,=}$ (βM_t / (As_b)

FAILURE DUE TO WEAR STRESS

The allowable stress in wear is

$s_w = S_w$ / F.S.

The relationship between induced wear stress and tooth thickness is

$$s_w=\gamma\sqrt{\left(\bar{M}_t/\bar{t}\right)/\bar{A}}$$

where

$$\gamma=\frac{0.59+(i+1)\sqrt{(i+1\_EK_cK_d}}{i\sqrt{\sin2\alpha}}$$

The face width due to wear is calculated as

t_2 = (γ/A)^2 ($M_t/s_w^{\,2}$)

The greater of t_1, t_2 is taken as the face width of the gear pair.

RESULTS

The values of the face widths of the gear pairs are calculated using a JAVA program.

1 The power is taken as 10 HP and the coefficient of variation of speed is varied from 0.01 to 0.1. The face widths obtained for these values are tabulated and also represented by graph.

2 The coefficient of variation of speed is kept as constant (C_{nw}=0.1) and the power is varied from 2.5 HP to 20 HP. The face widths obtained for these values are tabulated and also represented by graph.

3 The coefficient of variation of speed and the power are kept as constant (C_{nw}=0.1, P = 10 HP) and the probability of failure is varied from $1x10^{-1}$ to $1x10^{-6}$ and the face widths obtained for these values are tabulated and also represented by graph.

4 Face width of gear pairs with variation of C_{nw}:

5 The face widths of gear pairs in a 6 speed gear box obtained by varying C_{nw} from 0.01 to 0.1 are tabulated. The results obtained in deterministic design are also shown in the Table 1.

6 Input given: Pressure angle=20 degrees; Power=10HP; Z=2.574899

The face widths of gear pairs in a 6 speed gear box obtained by varying power from 2.5 HP to 20 HP are tabulated in Table 5.5.

Input given:Pressure angle=20 degrees; Z=2.574899; C_{nw}=0.1

The face widths of gear pairs in a 6 speed gear box obtained by varying the probability of failure from $1x10^{-1}$to$1x10^{-6}$ are tabulated.

Input given: Pressure angle=20 degrees; power=10HP; C_{nw}=0.1

GRAPHS: FOR 6 SPEED GEAR BOX

The Graph 1 shows the effect of horse power on face width of gear pairs. Face width is taken as Y-axis and power is taken as X-axis. It is clear that from the graph, face widths of gear pairs increases with increase in power.

The Graph 2 shows the effect of reliability on face width of gear pairs. Face width is taken as Y-axis and Reliability is taken as X-axis. It is evident that the face widths of gear pairs increase with increase in reliability.

Table 1: Face Width of Gear Pairs with Variation of Cnw

GEAR	COEFFICIENT OF VARIATION C_{nw}						Deterministic
PAIR	0.01	0.03	0.05	0.07	0.09	0.1	design(FS=2)
1.(G1,G4)	0.6641	0.6663	0.6707	0.6772	0.6857	0.6907	1.397
2.(G2,G5)	0.9080	0.9110	0.9170	0.9259	0.9375	0.9443	1.9099
3.(G3,G6)	0.5381	0,53980	0.5434	0.5487	0.5556	0.5596	1.132
4.(G7,G9)	0.7052	0.7087	0.7156	0.7255	0.7379	0.7450	1.4109
5.(G8,G10)	1.6431	1.6486	1.6594	1.6755	1.6965	1.7088	3.456

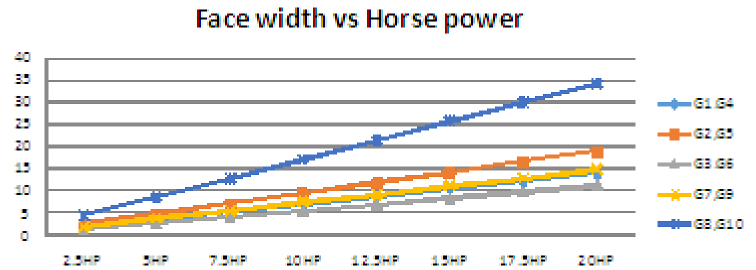
Table 2: Face Width of Gear Pairs with Variation of Power

GEAR PAIR	Power							
	2.5HP	5HP	7.5HP	10HP	12.5HP	15HP	17.5HP	20HP
1.(G1,G4)	0.1726	0.3453	0.5180	0.6907	0.8634	1.0361	1.2088	1.3814
2.(G2,G5)	0.2360	0.4721	0.7082	0.9443	1.1804	1.4165	1.6526	1.8887
3.(G3,G6)	0.1399	0.2798	0.4197	0.5596	0.6995	0.8394	0.9793	1.1192
4.(G7,G9)	0.1862	0.3725	0.5587	0.7450	0.9313	1.1175	1.3038	1.4900
5.(G8,G10)	0.4272	0.8544	1.2816	1.7088	2.1361	2.5633	2.9905	3.4177

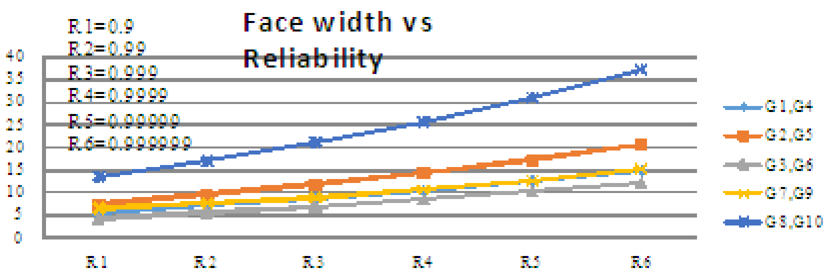
Table 3: Face Widths of Gear Pairs with Variation of Probability of Failure

GEAR PAIR	Probability of Failure					
	$1X10^{-1}$	**$1X10^{-2}$**	**$1X10^{-3}$**	**$1X10^{-4}$**	**$1X10^{-5}$**	**$1X10^{-6}$**
1.(G1,G4)	0.5298	0.6907	0.8573	1.0421	1.2535	1.5057
2.(G2,G5)	0.7243	0.9443	1.1721	1.4247	1.7138	2.0586
3.(G3,G6)	0.4292	0.5596	0.6946	0.8443	1.0156	1.2199
4.(G7,G9)	0.6363	0.7450	0.8658	1.0524	1.2660	1.5207
5.(G8,G10)	1.3597	1.7088	2.1210	2.5781	3.1012	3.7251

Graph 1: Effect of Horse Power on Face Width of Gear Pairs



Graph 2: Effect of Reliability on Face Width of Gear Pairs



CONCLUSION

1. Reliability based design procedure is more realistic and useful.
2. In the present work, normal distribution for the variables is assumed. Even, if the variables follow other than normal distribution, the basic approach applied here will not change.
3. The probabilistic design procedure is quite general and can be applied to any machine element.
4. Over designing of the elements can be avoided with the help of probabilistic design.
5. It is evident from the above results that as the reliability increases face width of the gear pair also increases.
6. As the coefficient of variation of speed increases, the face width of all gears increases.
7. As the input power increases face widths of the gear pairs increase linearly.
8. It is evident from the results that as the factor of safety increases face width of the gear pair and mass of the gear box increases.

The reliability based optimum design results are obtained with three different C1 and C2 values. It can be observed that all combinations of C1 and C2 yield essentially the same optimum face width and minimum mass of gear box.

9. The present result indicates that the minimum mass of the gear box will be higher in the case of probabilistic design compared to that of the deterministic design.

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