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Research Paper

NUMERICAL STUDY OF ANTI-SYNCHRONIZATION OF A NATURAL SATELLITE (ENCELADUS)

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we study the anti-synchronization (AS) behaviour of a chaotic rotational dynamics of enceladus (natural satellite of Saturn) with another identical dynamical system evolving from different initial conditions using the active control technique. The designed controllers, with our own choice of the coefficient matrix of the error dynamics, are found to be effective in the stabilization of the error states at the origin. Numerical simulations are presented to illustrate the effectiveness of the proposed active control technique using mathematica.

Keywords: Chaos, Anti-synchronization (AS), Enceladus

INTRODUCTION

During the so far progress in the synchronization chaotic systems, linear Active and the Non-linear Control methods are the one that are widely implemented to synchronize the identical as well as non-identical chaotic systems from various fields of nonlinear sciences (G Cai and Z Tan, 2007; H Zhang and X K Ma, 2004; A B Saaban *et al*., 2013; 2014; I Ahmad *et al*., 2014a; 2014b; 2014c).

A phenomenon similar to synchronization in which the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the master system known as AS. In this phenomenon the sum of two signals is expected to converge to zero when AS appears. Recently, active control has been extensively implemented to study the AS behavior of two identical as well as non-identical chaotic systems (Y Zhang and J Sun, 2004; J B Liu *et al*., 2000; M Shahzad, 2011a; M C Ho *et al*., 2002; M Shahzad, 2011b; U E Vincent and A Ucar, 2007; U E Vincent and J A Laoye, 2007; C M Kim, 2003; A A Emadzadeh and M Haeri, 2005; C Li and X Liao, 2006). In fact, in engineering, it is hardly the case that every component can be assumed to be identical. Thus, it is much more attractive and challengeable to realize AS of two different chaotic systems. Motivated by the afore mentioned studies, we aim to implement the active control method to study the AS behavior of the rotational dynamics of enceladus with another identical dynamical system evolving from different initial conditions.

In our present paper, we continue our study to use the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria to study the AS behavior of the rotational dynamics of enceladus (M Shahzad, 2011b) that is out of round satellite in a fixed elliptical orbit with spin axis perpendicular to the orbit plane with another identical dynamical system evolving from different initial conditions.

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ANTI-SYNCHRONIZATION VIA ACTIVE CONTROL

For a system of two coupled chaotic oscillators, the master system ($\dot{x}_i = f(x_i, y_i)$ for $i = 1,2,3$) and the slave system ($\dot{y}_i = g(x_i, y_i)$ for $i = 1,2,3$), where $x_i(t)$ and $y_i(t)$ are the phase space (state variables), $f(x_i, y_i)$ and $g(x_i, y_i)$ and are the corresponding nonlinear functions, AS in a direct sense implies $\lim_{t\to\infty}|x_i(t)+y_i(t)| \to 0$ for $i=1,2,3$. When this occurs the coupled systems are said to achieve AS completely. This phenomenon has been investigated both experimentally and theoretically in many physical systems (G Cai and S Zheng, 2008; M Shahzad, 2012a; A Khan and M Shahzad, 2012; M Shahzad, 2012b; M Shahzad and I Ahmed, 2013; M Shahzad and M Raziuddin, 2013a; 2013b; 2013c; M Shahzad and M Haris, 2014).

In order to formulate the active controllers for AS, we need to redefine the error functions as $e_i(t) = y_i(t) + x_i(t)$ for $i=1,2,3$, where $e_i(t)$ for $i=1,2,3$ are called the AS errors such that $\lim_{t\to\infty} e_i(t) \to 0$ for $i=1,2,3$.

We recall the master (driver) and slave (response) system from our previous study (M Shahzad, 2012b) to extend our study for AS behavior of the rotational dynamics of Enceladus using active control.

Master system:

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \frac{\varepsilon^2 n^2 e}{4}\sin(x_1+x_3) - \frac{7\varepsilon^2 n^2 e}{4}\sin(3x_3 - x_1) - \frac{\varepsilon^2 n^2}{2}\sin 2x_3,$$

$$\dot{x}_3 = x_2 - n.$$

(1.1)

Slave system:

$$\dot{y}_1 = y_2 + u_1(t),$$

$$\dot{y}_2 = \frac{\varepsilon^2 n^2 e}{4}\sin(y_1+y_3) - \frac{7\varepsilon^2 n^2 e}{4}\sin(3y_3 - y_1) - \frac{\varepsilon^2 n^2}{2}\sin 2y_3 + u_2(t),$$

$$\dot{y}_3 = y_2 - n + u_3(t).$$

Where $\varepsilon = \sqrt{3(B-A)/C}$ the moments of inertia are $A < B < C$, n is the orbital frequency, e is the eccentricity of the satellite, $x_i(t)$ & $y_i(t)$ are the state variables of master & slave systems respectively and $u_i(t)$ for $i = 1,2,3$, are control functions to be determined. Let $e_i(t) = y_i(t) + x_i(t)$ for $i = 1,2,3$ be the AS errors such that in AS state $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1,2,3$. From (1.1) and (1.2), we obtain the error dynamics.

$$\dot{e}_1 = e_2 + u_1(t),$$

$$\dot{e}_2 = \frac{\varepsilon^2 n^2 e}{4}\{\sin(y_1+y_3)+\sin(x_1+x_3)\} - \frac{7\varepsilon^2 n^2 e}{4}\{\sin(3x_3-x_1)+\sin(3y_3-y_1)\}$$
$$-\frac{\varepsilon^2 n^2}{2}\{\sin 2x_3 + \sin 2y_3\} + u_2(t),$$

$$\dot{e}_3 = e_2 - 2n + u_3(t).$$

(1.3)

The error system (1.3) to be controlled is a linear system with control inputs. Therefore the control functions can be redefined in order to eliminate the terms in (1.3) which cannot be expressed as linear terms in e_1, e_2 and e_3 as follows:

$$u_1(t) = v_1(t),$$

$$u_2(t) = -\frac{\varepsilon^2 n^2 e}{4}\{\sin(y_1+y_3)+\sin(x_1+x_3)\} + \frac{7\varepsilon^2 n^2 e}{4}\{\sin(3x_3-x_1)+\sin(3y_3-y_1)\}$$
$$+\frac{\varepsilon^2 n^2}{2}\{\sin 2x_3 + \sin 2y_3\} + v_2(t),$$

$$u_3(t) = v_3(t) + 2n.$$

(1.4)

Therefore the linear error system can be written as follows:

$$\dot{e}_1(t) = e_2(t) + v_1(t),$$

$$\dot{e}_1(t) = v_2(t),$$

$$\dot{e}_3(t) = e_2(t) + v_3(t).$$

(1.5)

The error system (1.5) to be controlled is a linear system with control inputs $v_1(t)$, $v_2(t)$, and *v3(t)* as the function of the error states $e_1(t)$, $e_2(t)$ and $e_3(t)$. As stated, as long as $\lim_{t\to\infty} e_i(t) \to 0$ for $i=1,2,3$, AS between the master and slave system is realized, that is, the system represented by (1.1) and (1.2) are achieving AS under active control. According to active control method, the controllers v_1, v_2, and $v_3$can be written as:

$$(v_1 \quad v_2 \quad v_3)^T = A(e_1 \quad e_2 \quad e_3)^T$$

(1.6)

Where A is a 3x3 constant matrix. As per the Lyapunov stability theory and the Routh-Hurwitz criteria, in order to make the close loop system (1.5) stable, the proper choice of the elements of A is such that the system (1.6) must have all eigen values with the negative real parts:

Let $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$, bring (1.6) into (1.5), we may obtain

$$(\dot{e}_1 \quad \dot{e}_2 \quad \dot{e}_3)^T = B(e_1 \quad e_2 \quad e_3)^T$$

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + A\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Now the slave system (1.2) can be defined as

$$\dot{y}_1 = 2y_2 - y_1 + x_2 + x_1,$$

$$\dot{y}_2 = \frac{-\varepsilon^2 n^2 e}{4}\sin(x_1 + x_3) + \frac{7\varepsilon^2 n^2 e}{4}\sin(3x_3 - x_1) + \frac{\varepsilon^2 n^2}{2}\sin 2x_3 - x_2 - y_2,$$

$$\dot{y}_3 = 2y_2 - x_3 + x_2 - y_3 + n.$$

(1.7)

NUMERICAL SIMULATION FOR ANTI-SYNCHRONIZATION

For the parameters involved in system under investigation, $e = 0.45$, $\varepsilon = 0.44$, and $n = 0.3$ and the initial conditions for master & slave systems $x_1(0) = 0$, $x_2(0) = 0.1$, $x_3(0) = 0.3$, $y_1(0) = 0.2$, $y_2(0) =$ 0.4 and $y_3(0) = 0.6$ and respectively, the system has been simulated using mathematica. The obtained results show that the system under consideration achieved AS. In Figure 1, we have plotted the time series of the errors where as in Figure 2-4, the time series of the states variables of master & slave systems have been plotted with & without controller. Further, it also has been confirmed by the convergence of the AS quality defined by

$$e(t) = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)}$$

(2.1)

Figure (6) confirms the convergence of the errors of AS quality defined by (2.1).

Figure 1: Time Series of e_1, e_2 & e_3

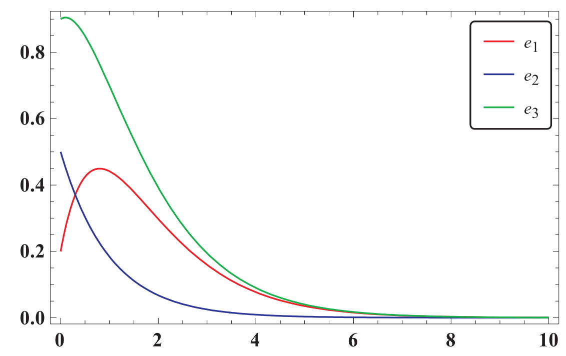


Figure 2: Time Series of x_1 & y_1

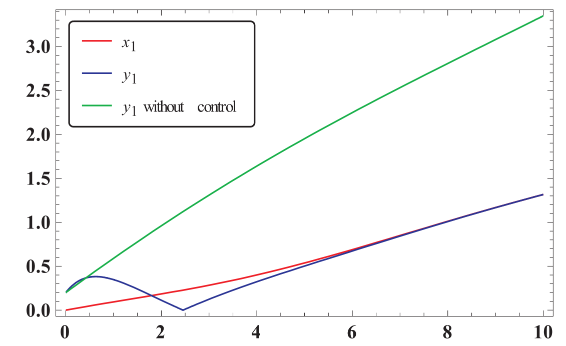


Figure 3: Time Series of x_2 & y_2

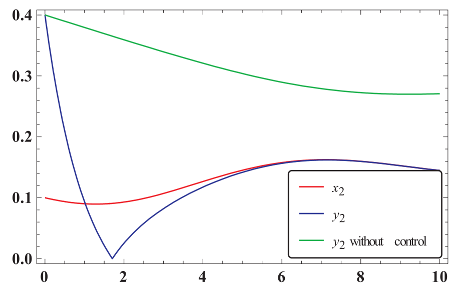


Figure 4: Time Series of x_3 & y_3

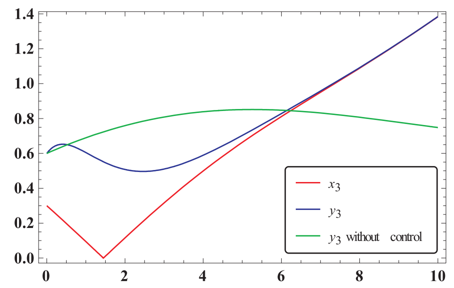


Figure 5: Time Series of V(t)

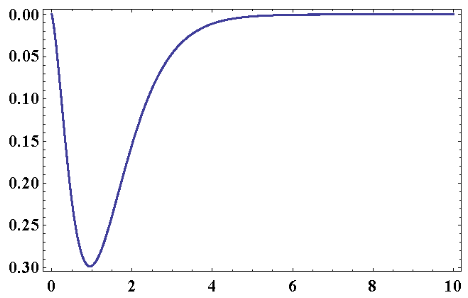
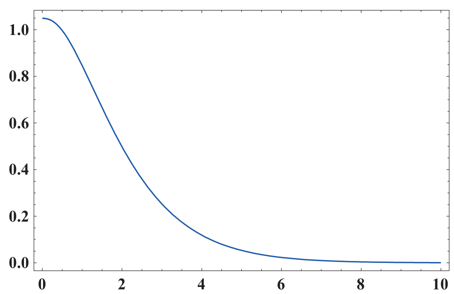


Figure 6: Convergence of Errors



CONCLUSION

In this computational study, we have investigated AS behavior of the rotational dynamics of enceladus (natural satellite of Saturn) that is out of round satellite in a fixed elliptical orbit with spin axis perpendicular to the orbit plane with another identical dynamical system evolving from different initial conditions via the active control technique. The simulated results show that the system under consideration has achieved AS which is strongly guaranteed by all the plotted graphs.

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